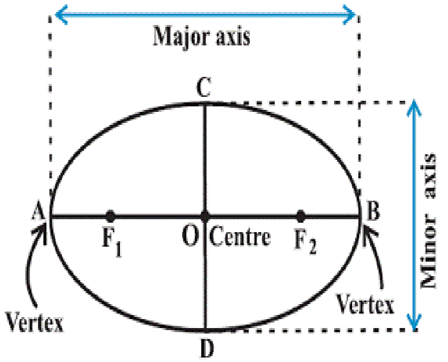
ELLIPSE

Created: Monday, 12 September 2011 08:45 | Published: Monday, 12 September 2011 08:45 | Written by Super User | Print

Introduction to Ellipses

An ellipse is a plane curve that results from the intersection of a cone by a plane in a way that produces a closed curve. Circles are special cases of ellipses, obtained when the cutting plane is orthogonal to the cone's axis. An ellipse is also the locus of all points of the plane whose distances to two fixed points add to the same constant.

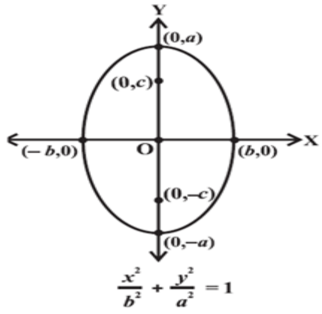
Elements of an Ellipse

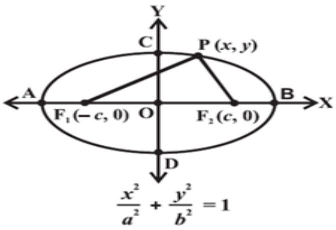


The two fixed points F_1 and F_2 are called the foci (plural of ‘focus’). The midpoint of the line segment joining the foci is called the centre of the ellipse. The line segment through the foci of the ellipse is called the major axis and the line segment through the centre and perpendicular to the major axis is called the minor axis. The end points of the major axis are called the vertices of the ellipse.

Suppose the lengths of the major axis and n=minor axis is 2a and 2b respectively. Also, the distance between the foci be 2c. Thus, the length of the semi major axis is ‘a’ and semi-minor axis is ‘b’.

Equations of an ellipse



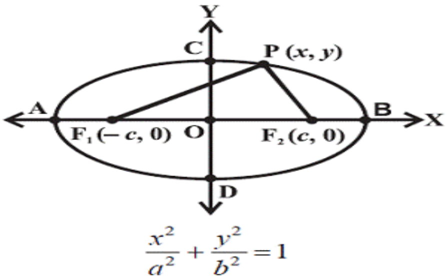


Above diagrams represent two possible orientations of ellipse.

Now, we will derive theequation for the ellipse with foci on the x – axis.

Let F_1 and F_2 be the foci and O be the midpoint of the line segment F_1F2. Let O be the origin and the line from O through F_2 be the

positive x-axis and that through F_1 as the negative x-axis.



Let, the line through O perpendicular to the x-axis is the y-axis. Let the coordinates of F_1 be (– c, 0) and F_2 be (c, 0).

Let P(x, y) be any point on the ellipse such that the sum of the distances from P to the two foci be 2a so given

$PF_1 + PF_2 = 2a$

Applying distance formula in above equation:
$?((x + c)^2 + y^2) + ?((x – c)^2 + y^2)= 2a$

$?((x + c)^2 + y^2) = 2a - ?((x – c)^2 + y^2)$

Squaring both sides:
$(x + c)^2 + y^2 = 4a^2 – 4a?((x – c)^2 + y^2) + (x + c)^2 + y^2$

On further simplification:
$?((x - c)^2 + y^2) = a – cx/a$

On further squaring and simplifying:
$x^2/a^2 + y^2/(a^2 – c^2) = 1$

$x^2/a^2 + y^2/b^2 = 1$

Hence, any point on the ellipse satisfies:
$x^2/a^2 + y^2/b^2 = 1$

To summarise, here are the observations from the standard equations of ellipse:

1. Ellipse is symmetric with respect to both the coordinate axes since if (x, y) is a point on the ellipse, then (– x, y), (x, –y) and (– x, –y) are also points on the ellipse.

2. The foci always lie on the major axis. The major axis can be determined by finding the intercepts on the axes of symmetry. That is, major axis is along the x-axis if the coefficient of x^2 has the larger denominator and it is along the y-axis if the coefficient of y^2 has the larger denominator.

Eccentricity

The eccentricity for the ellipse $x^2/a^2 + y^2/b^2 = 1$,

We have:
$b^2 = a^2 (1 – e^2)$

$e^2 = 1 – b^2/a^2$

$e^2 = 1 - 4b^2/4a^2$

$e^2 = 1 – (2b)^2/(2a)^2$

e = ?(1 – (Minor Axis/Major Axis)^2)

Latus Rectum

Latus rectum of an ellipse is a line segment perpendicular to the major axis through any of the foci and whose end points lie on the ellipse.

The length of the latus rectum of ellipse $x^2/a^2 + y^2/b^2 = 1$ is $2b^2/a$.

Now try it yourself! Should you still need any help,click here to schedule live online session with e Tutor!

Reference Links:

- http://en.wikipedia.org/wiki/Cone_(geometry)
- http://en.wikipedia.org/wiki/Circle
- http://en.wikipedia.org/wiki/Locus_(mathematics)
- http://en.wikipedia.org/wiki/Ellipse#Elements_of_an_ellipse
- http://en.wikipedia.org/wiki/Ellipse#Equations
- http://www.thefreedictionary.com/eccentricity
- http://www.answers.com/topic/latus-rectum

Category:ROOT