HYPERBOLA

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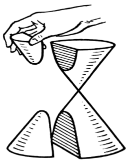
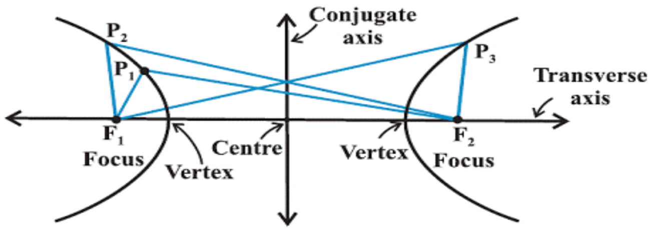
Introduction to Hyperbolas

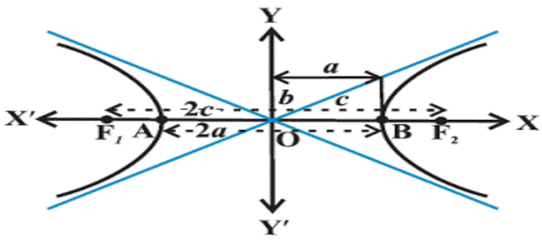


A hyperbola is an open curve with two branches, the intersection of a plane with both halves of a double cone.



$$P_1F_2 - P_1F_1 = P_2F_2 - P_2F_1 = P_3F_1 - P_3F_2$$

The two fixed points F1 and F2 are called the foci of the hyperbola. The midpoint of the line segment joining the foci is called the centre of the hyperbola. The line through the foci is called the transverse axis and the line through the centre and perpendicular to the transverse axis is called the conjugate axis. The points at which the hyperbola intersects the transverse axis are called the vertices of the hyperbola.



In the above figure, let the distance between the two foci F_1 and F_2 be 2c, and the distance between the two vertices A and B be 2a. With the assumed information, we can define b as

b = ?$c^2 - a^2$

Also 2b is the length of the conjugate axis.

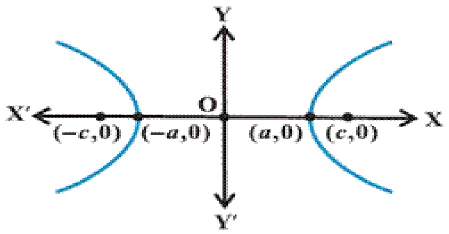
By taking the point P at A and B in the above figure, we have

$BF_1 - BF_2 = AF_2 - AF_1$ (By definition)

$BA + AF_1 - BF_2 = AB + BF_2 - AF_1$

i.e., $AF_1 = BF_2$

So that, $BF_1 - BF_2 = BA + AF_1 - BF_2 = BA = 2a$



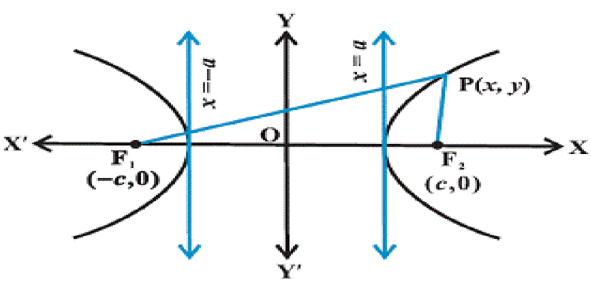
$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

Equation of Hyperbola

Above diagrams represent two possible orientations of hyperbola.

Now, we will derive theequation for the hyperbola with foci on the x - axis. Consider the following diagram:



Let F_1 and F_2 be the foci and O be the mid-point of the line segment F_1F_2. Let O be the origin and the line through O via F_2 be the positive x – axis and that through F_1 as the negative x – axis. Let the coordinates of F_1 be (-c, 0) and F_2 be (c, 0).

Let P(x, y) be any point on the hyperbola such that the difference between the farther and closer point be 2a

? $PF_1 – PF_2 = 2a$ - (i)

Applying distance formula in the above equation:
$?(x + c)^2 + y^2 - ?(x - c)^2 + y^2 = 2a$

$?(x + c)^2 + y^2 = 2a + ?(x - c)^2 + y^2$

On squaring both sides, we get:
$(x + c)^2 + y^2 = 4a^2 + 4a?(x - c)^2 + y^2 + (x – c)^2 + y^2$

On further simplifying:
$cx/a – a = ?(x - c)^2 + y^2$

On squaring both sides again and simplifying further:
$x^2/a^2 – y^2/(c^2 – a^2) = 1$

$x^2/a^2 – y^2/b^2 = 1$ (Since $c^2 – a^2 = b^2$)

Hence, any point on the hyperbola satisfies

$\mathbf{x^2/a^2 – y^2/b^2 = 1}$

To summarise, here are the observations from the standard equations of parabola:

1. Hyperbola is symmetric with respect to both the axes, since if (x, y) is a point on the hyperbola, then (– x, y), (x, – y) and (– x, – y) are also points on the hyperbola.

2. The foci are always on the transverse axis. It is the positive term whose denominator gives the transverse axis.

Eccentricity

For the hyperbola, $\mathbf{x^2/a^2 – y^2/b^2 = 1}$

We have, $b^2 = a^2 (e^2 – 1)$

$e^2 = (a^2 + b^2)/a^2$

$e^2 = 1 + b^2/a^2$

$e = ?1 + b^2/a^2$

$e = ?1 + (2b)^2/(2a)^2$

e = ?1 + (Conjugate axis)^2/(Transverse axis)^2

Latus Rectum

Latus rectum of hyperbola is a line segment perpendicular to the transverse axis through any of the foci and whose end points lie on the hyperbola.

Length of latus rectum in hyperbola is $2b^2/a$

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Reference Links:

- http://en.wikipedia.org/wiki/Hyperbola
- http://www.answers.com/topic/conjugate-axis
- http://www.answers.com/topic/transverse-axis
- http://en.wikipedia.org/wiki/Vertex_(geometry)
- http://en.wikipedia.org/wiki/Hyperbola#In_Cartesian_coordinates

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