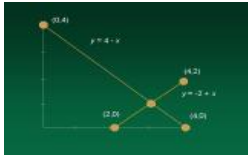


# Vector and Cartesian Equations of a Line

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## Equation of a line in space

We have studied equation of lines in previous classes. Now we will learn the vector and Cartesian [equation of a line](#) in space.

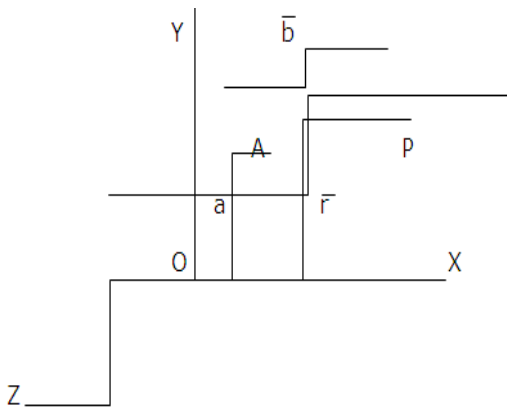


A line is uniquely determined if,

- It passes through a given point and has given direction
- It passes through two given points.

## Equation of a line through a given point and parallel to a given vector

Let  $\vec{a}$  be given [position vector](#) of the given point and  $\vec{b}$  be the given vector, then its equation is given by  $\vec{r} = \vec{a} + \lambda \vec{b}$



## Vector Equation

In the above figure  $\overrightarrow{AP}$  is parallel to  $\overrightarrow{b}$ , so  $\overrightarrow{AP} = \lambda \overrightarrow{b}$  .....(1)

$$\overrightarrow{AP} = \overrightarrow{OP} - \overrightarrow{OA}$$

$$= \vec{r} - \vec{a}$$

(1) becomes,  $\vec{r} - \vec{a} = \lambda \overrightarrow{b}$

$$\vec{r} = \vec{a} + \lambda \overrightarrow{b}, \text{ which is the vector equation.}$$

Hence [vector equation](#) of a line passing through a point with position vector  $\vec{a}$  and parallel to a given vector  $\vec{b}$  is given by  $\vec{r} = \vec{a} + \lambda \overrightarrow{b}$

## Cartesian Equation

Let the coordinates of the given point be  $A(x_1, y_1, z_1)$  and the direction ratios of the parallel vector be  $\langle a, b, c \rangle$ . Let  $P(x, y, z)$  be any point (General point) on the line.

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$$

The Cartesian equation is given by

**Example:** Find the vector and Cartesian equations of the line through the point  $(5, 3, -5)$  and which is parallel to the vector  $4\hat{i} - 7\hat{j} + 3\hat{k}$

**Solution:** We have  $\vec{a} = 5\hat{i} + 3\hat{j} - 5\hat{k}$  and  $\vec{b} = 4\hat{i} - 7\hat{j} + 3\hat{k}$ , so

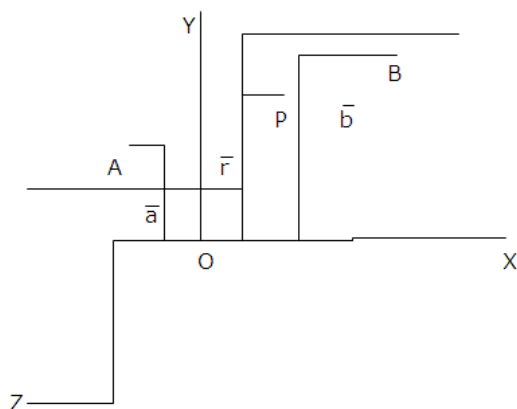
Vector equation is  $\vec{r} = \vec{a} + \lambda \vec{b}$

$$= (5\hat{i} + 3\hat{j} - 5\hat{k}) + \lambda (4\hat{i} - 7\hat{j} + 3\hat{k})$$

$$\text{Cartesian Equation is } \frac{x-5}{4} = \frac{y-3}{-7} = \frac{z-(-5)}{3}$$

## Equation of a line passing through two points

Let  $\vec{a}$  and  $\vec{b}$  be the position vectors of two points that are lying on a given line then their equation is given by  $\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$



## Vector Equation

Let  $\vec{a}$  and  $\vec{b}$  be the position [vectors](#) of the points lying on the line and  $\vec{r}$  be the position of any point (general point).

We know  $\vec{AP}$  and  $\vec{AB}$  are collinear vectors, therefore P will lie on the line if and only if  $\vec{AP} = \lambda \vec{AB}$

$$\vec{r} - \vec{a} = \lambda (\vec{b} - \vec{a})$$

$$\vec{r} = \vec{a} + \lambda (\vec{b} - \vec{a}), \text{ which is the vector equation.}$$

## Cartesian Equation

Let  $A(x_1, y_1, z_1)$  and  $B(x_2, y_2, z_2)$  be two point in the line and  $P(x, y, z)$  be a general point on the line, the Cartesian Equation is given by

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

**Example:** Find the Vector and Cartesian equation of the line joining the points  $(-1, 3, 2)$  and  $(3, 0, 1)$

**Solution:** Here  $\vec{a} = -\hat{i} + 3\hat{j} + 2\hat{k}$  and  $\vec{b} = 3\hat{i} + 0\hat{j} + \hat{k}$

Vector equation is  $\vec{r} = (-\hat{i} + 3\hat{j} + 2\hat{k}) + \lambda(4\hat{i} - 3\hat{j} - \hat{k})$

Cartesian equation is

$$\frac{x - (-1)}{4} = \frac{y - 3}{-3} = \frac{z - 2}{-1}$$

Now try it yourself! Should you still need any help, [click here](#) to schedule live online session with e Tutor!

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## Reference links:

- [http://en.wikipedia.org/wiki/Position\\_%28vector%29](http://en.wikipedia.org/wiki/Position_%28vector%29)
- <http://www.revisesmart.co.uk/maths/core-4/vector-equation-of-a-line.html>
- <http://en.wikiversity.org/wiki/Vectors>

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