ASYMPTOTIC AND UNBOUNDED BEHAVIOUR

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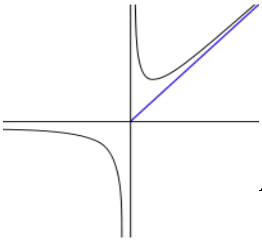
Asymptotes



An asymptote of a curve is a line such that the distance between the curve and the line approaches zero as they tend to infinity. Mainly there are three types of asymptotes horizontal, vertical and oblique. The notion of asymptote is related to the procedure of curve sketching. It serves as a guideline and helps to show the behavior of the curve towards infinity.

For example the asymptotes to thehyperbola $(x^2/a^2) – (y^2/b^2) = 1$ is given by the equations $y = \pm (b/a)x$

Types of Asymptotes

We have the following types of asymptotes:

a) Horizontal Asymptotes
b) Vertical Asymptotes
c) Oblique Asymptotes

Let’s discuss each one of them in detail:

Horizontal Asymptotes

Let y = f(x) be a function. Suppose that L be a number such that whenever ‘x’ is large f(x) is close to L and let f(x) be made very close to L by making x larger. Then we say that the limit of f(x) as x approaches to +? is L and we write it as

lim f(x) = L
x — ?

Similarly suppose that M is a number such that whenever ‘x’ is a large negative number, f(x) is close to M and let f(x) be made very close to M by making x a larger negative number. Then we say that the limit of f(x) as x approaches to -? is M and we write it as

lim f(x) = M
x—-?

In both the cases we refer to y = L and y= M as horizontal asymptotes of the function f.

Example: Find the horizontal asymptote of y = x + 2

$x^2 + 1$

Solution: Given $y = f(x) = \frac{x + 2}{x^2 + 1}$

$$\lim_{x \to ?} f(x) = \lim_{x \to ?} \frac{x + 2}{x^2 + 1}$$

$$= \lim_{x \to ?} \frac{x[1 + (2/x)]}{x^2[1 + (1/x^2)]}$$

$$= \lim_{x \to ?} \frac{[1 + (2/x)]}{x[1+ (1/x^2)]}$$

$$= \frac{1 + 0}{?\,(1 + 0)}$$

$$= 0$$

Hence the horizontal asymptote is $y = 0$.

Vertical Asymptotes

Vertical asymptotes are vertical lines which correspond to the zeroes of the denominator of a rational function. Vertical asymptotes and domain are related to each other. Domain is the set of values given to 'x' whereas vertical asymptote is obtained by putting denominator equal to zero. We can have some examples.

1. Find the domain and vertical asymptotes for the following function

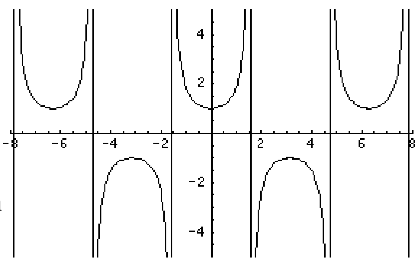
$$f(x) = \frac{x + 2}{x^2 + 2x - 8}$$

Solution: Consider the denominator $x^2 + 2x - 8=0$.

On factorizing we get, $(x + 4)(x - 2) = 0$

$x = -4, 2$

Hence domain



: x ? $-4, 2$

Vertical Asymptotes are $x = -4, 2$

Also keep in mind that trigonometric functions can go to zero repeatedly, so for the trigonometric function y=sec(x) there are many vertical asymptotes. All the vertical lines shown below in the figure are the vertical asymptotes.

Oblique asymptotes

When a linear asymptote is not parallel to x or y axes, then we say it is oblique or slanting asymptote. When the numerator of a rational function has degree exactly one more than the denominator, the function has oblique asymptote. This is because when dividing the fraction, there will a linear term and the remainder.

For example, consider the function

$$f(x) = \frac{x^2 + 3x + 2}{x - 2}$$

On dividing the polynomial, we get

$$\frac{x^2 + 3x + 2}{x - 2} = (x+5) + \frac{12}{x - 2}$$

The oblique asymptote is the linear part and not the remainder, so it is given by y = x + 5

Hence the oblique or slant asymptote is y = x+5

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Reference Links:

- http://en.wikipedia.org/wiki/Asymptote
- http://en.wikipedia.org/wiki/Hyperbola
- http://cnx.org/content/m13606/latest/
- http://archives.math.utk.edu/visual.calculus/1/vertical.4/index.html
- http://www.cs.gmu.edu/cne/modules/dau/calculus/limits/limits8_bdy.html

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