## DIFFERENTIATION - I

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## Introduction



In this topic, we are dealing with different types of differentiation. Different methods are derivatives of implicit functions, exponential functions, and logarithmic functions.

## Derivatives of Implicit Functions

When a relationship between x and y is expressed in a way that it is easy to solve for y and write $\mathrm{y}=\mathrm{f}(\mathrm{x})$, we say that y is given as an explicit function of ' $x$ ', otherwise it is implicit that $y$ is a function of $x$ and we say that the relationship gives function implicitly. For example: 1) $2 x+3 y+5=0$
2) $x+\cos x y+3=0$

In the above examples, first one is an explicit function, since it can be easily written as $y=(-2 x-5) / 3$ and is easy to solve for $y$. But in the second one, we find difficult to separate $y$ from that function, so it is an implicit function. In the case of an implicit function, we directly differentiate the given function with respect to x and find $\mathrm{dy} / \mathrm{dx}$.
Find $d y / d x$ for the function $a x+b y^{2}=\operatorname{Cos} y$
Differentiating with respect to x we get,
a. $1+$ b. $2 y d y / d x=-\sin y d y / d x$
$(2 b y+\sin y) d y / d x=-a$
$d y / d x=-a$
$2 b y+\sin y$

## Exponential Function

The exponential function with positive base $\mathrm{b}>1$ is the function $\mathrm{y}=\mathrm{b}^{\mathrm{x}}$
The domain of the exponential function is $R$, the set of real numbers and the range is set of all positive real numbers.
Exponential function with base 10 is called the common exponential function, which is given by $y=10^{\mathrm{x}}$
Using 'e' as the base, we get an extremely important exponential function $\mathrm{y}=\mathrm{e}^{\mathrm{x}}$ which is called natural exponential function
The derivative of $e^{x}$ is $e^{x}$ itself

$$
\frac{d\left(e^{x}\right)}{d x}=e^{x}
$$

x
The derivative of $b$ is given by,

## Logarithmic Function

Let $b>1$ and if $b^{x}=a$, then we say logarithm of ' $a$ ' to the base ' $b$ ' is ' $x$ ' and it is denoted by $\log _{b} a=x$. This function is defined from $R^{+}$ to R.
The domain of logarithmic function is $\mathrm{R}^{+}$and the range is R , set of all real numbers.
If the base $b=10$, then we say it as common logarithm and if $b=e$ then it is natural logarithm.
The derivative of $\log$ function is given by

$$
\frac{d(\log x)}{d x}=\frac{1}{x}
$$

## Logarithmic Differentiation

Logarithmic differentiation is applicable for the functions in the form
$\mathrm{y}=[\mathrm{u}(\mathrm{x})]^{\mathrm{v}(\mathrm{x})}$, taking log on both sides we get,

$$
\begin{aligned}
\log y & =\log [u(x)]^{v(x)} \\
& =v(x) \log [u(x)]
\end{aligned}
$$

Using Chain Rule, we may differentiate this to get,

$$
\begin{aligned}
\frac{1}{y} \frac{d y}{y d x} & =v(x) \frac{1}{u(x)} u^{\prime}(x)+v^{\prime}(x) \log [u(x)] \\
\frac{d y}{d x} & =y\left(\frac{v(x)}{u(x)} u^{\prime}(x)+v^{\prime}(x) \log [u(x)]\right)
\end{aligned}
$$

The main point to be noted in this method is that ' $y$ ' and $u(x)$ must always be positive otherwise their logarithms are not defined. This process of differentiation is known as Logarithmic Differentiation.

## Laws of logarithms

1) $\log \mathrm{b} \mathrm{pq}=\log \mathrm{p} \mathrm{p}+\log \mathrm{bq}$
2) $\log (\mathrm{p} / \mathrm{q})=\log _{\mathrm{b}} \mathrm{p}-\log _{\mathrm{b}} \mathrm{q}$
3) $\log _{b} p^{q}=q \log \mathrm{p} p \quad$ [Power Rule]
4) $\log _{a} p=\log _{b} p \quad$ [ Change of Base]
$\log _{b} a$
Example: Differentiate $\mathrm{x}^{\sin \mathrm{x}}, \mathrm{x}>0$ w.r.t x
Solution: Let $\mathrm{y}=\mathrm{x}^{\sin \mathrm{x}}$
Taking $\log$ on both sides,
$\log y=\log x^{\sin x}$
$\log y=\sin x \log x$
Differentiating,
$1 d y=\sin x 1+\log x \cos x$
$y d x \quad x$
$\left.\begin{array}{l}d y=y\left(\frac{\sin x}{x}+\cos x \log x\right. \\ d x\end{array}\right)$
$=x^{\sin x}(\sin x / x+\cos x \log x)$

Now try it yourself! Should you still need any help, click here to schedule live online session with e Tutor!

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## Reference Links :

- http://en.wikipedia.org/wiki/Implicit_and_explicit_functions
- http://en.wikipedia.org/wiki/Logarithmic differentiation
- http://en.wikipedia.org/wiki/Exponential function
- http://www.intmath.com/functions-and-graphs/2a-domain-and-range.php
- http://en.wikipedia.org/wiki/Logarithm
- http://en.wikipedia.org/wiki/Derivative

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