INDEFINITE INTEGRALS – I

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Methods of Integration

$$\int f(x)\,dx.$$

We have already discussed that integrals of some functions are obtained from their derivatives. It was based on inspection which means that on search of a function F whose derivative is 'f'. Method of inspection is not suitable for many of the functions. So we were forced to find additional methods to reduce a function to its standard form. The methods which we follow are

1) Integration using Partial Fractions
2) Integration by substitution
3) Integration by Parts.

Some Standard Integrals

1) ?x^n dx = $\frac{x^{n+1}}{n+1}$ + C , n?-1
2) ?dx = x + C
3) ?cosx dx = sinx + C
4) ?sinx dx = -cosx + C
5) ?sec^2x dx = tanx + C
6) ?$cosec^2$x dx = -cotx + C
7) ?secxtanx dx = secx + C
8) ?cosecxcotx dx = -cosecx + C
9) ?e^x dx = e^x + C
10) ?1/x dx = log|x| + C
11) ?dx/?(1-x^2) = sin^{-1}x + C
12) ?dx/?(1-x^2) = -cos^{-1}x + C
13) ?dx/(1+x^2) = tan^{-1}x + C
14) ?dx/(1+x^2) = -cot^{-1}x + C
15) ?dx/(x?x^2-1) = sec^{-1}x + C

Integration using Partial Fractions

We have already learned that a rational function is defined as the ratio of two polynomials in the form P(x)/Q(x), where P(x) and Q(x) are polynomials in 'x' and Q(x)?0. If the degree of P(x) is less than the degree of Q(x), then it is called proper otherwise improper.

The improper rational functions can be reduced to the proper rational functions by long division process. It can be expressed in the form

$$\frac{P(x)}{Q(x)} = T(x) + \frac{P1(x)}{Q(x)}$$

where T(x) is a polynomial in ‘x’ and P1(x)/Q(x) is a proper rational function. Here the rational functions which we consider for integration purpose will be those whose denominators can be factorized into linear and quadratic factors.
To evaluate ?P(x) dx, where P(x) is a proper rational function

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Q(x) Q(x)

we can express the integrand as a sum of simpler rational functions which is called the method of partial fractions.
After decomposing into simpler rational functions we can integrate it easily.

Different Forms of Partial Fractions

The following table indicates the types of simpler partial fractions that are to be associated with various kinds of rational functions. Before decomposing the rational functions we have to determine which form of partial fraction is to be applied.
Here A, B and C are constants (real numbers) to be determined by suitable method.

Sl. No:	Form of rational function	Form of the partial fraction
1	$\frac{px+q}{(x-a)(x-b)}$, $a \neq b$	$\frac{A}{(x-a)} + \frac{B}{(x-b)}$
2	$\frac{px+q}{(x-a)^2}$	$\frac{A}{(x-a)} + \frac{B}{(x-a)^2}$
3	$\frac{px^2+qx+r}{(x-a)(x-b)(x-c)}$	$\frac{A}{(x-a)} + \frac{B}{(x-b)} + \frac{C}{(x-c)}$
4	$\frac{px^2+qx+r}{(x-a)^2(x-b)}$	$\frac{A}{(x-a)} + \frac{B}{(x-a)^2} + \frac{C}{(x-b)}$
5	$\frac{px^2+qx+r}{(x-a)(x^2+bx+c)}$ where x^2+bx+c cannot be factorized.	$\frac{A}{(x-a)} + \frac{Bx+C}{(x^2+bx+c)}$

Find $\int \frac{2x\,dx}{x^2+3x+2}$

$$\int \frac{2x\,dx}{x^2+3x+2} = \frac{\int 2x\,dx}{(x+2)(x+1)}$$

Example: Take $\frac{2x}{(x+2)(x+1)} = \frac{A}{(x+2)} + \frac{B}{(x+1)}$ ....(1)

$2x = A(x+1) + B(x+2)$ [Taking L.C.M]
For finding A and B we have to assign values to ‘x’.
When x=-1, we get B=-2
When x=-2, we get A=4
(1) becomes

$$\frac{2x}{(x+2)(x+1)} = \frac{4}{(x+2)} + \frac{-2}{(x+1)}$$

$$\int\frac{2x\,dx}{(x+2)(x+1)} = \int\frac{4dx}{(x+2)} - \int\frac{2dx}{(x+1)}$$

$= 4 \log(x+2) – 2 \log(x+1) + C$

Integrals of some special function

When we come across the integrals like $\int\frac{dx}{ax^2+bx+c}$ and $\int\frac{dx}{\sqrt{(ax^2+bx+c}}$ where the denominator cannot be factorized as linear factors; we have to go for some other method. For solving such integrals we have to learn six formulas from which we get the answer easily.

The formulas are given below.

1) $\int\frac{dx}{x^2-a^2} = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + C$

2) $\int\frac{dx}{a^2-x^2} = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right| + C$

3) $\int\frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+C$

4) $\int\frac{dx}{\sqrt{x^2-a^2}} = \log|\,x+\sqrt{x^2-a^2}| + C$

5) $\int\frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}(\,x/a) + C$

6) $\int\frac{dx}{\sqrt{x^2+a^2}} = \log|x+\sqrt{x^2+a^2}| + C$

The remaining two methods of integration will be discussed in the article named "Indefinite Integrals – II".

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Reference Links:

- http://en.wikipedia.org/wiki/Partial_fractions_in_integration

- http://en.wikipedia.org/wiki/Rational_function
- http://en.wikipedia.org/wiki/Lists_of_integrals
- http://www.sosmath.com/algebra/pfrac/pfrac07.html

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