## LOCAL LINEAR APPROXIMATION

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## Introduction



In this section we will learn how derivatives can be used to approximate nonlinear functions by
simpler linear functions. We will also define the differentials dy and dx and use them to interpret the derivative dy/dx as a ratio of differentials.
Let a function ' f ' is differentiable at x 0 and recall that the equation of the tangent line to the graph of the function ' f ' through $\mathrm{P}\left(\mathrm{x}_{0}\right.$ , $f(x 0))$ is $y=f(x 0)+f$ ' ( $x 0)(x-x 0)$. Since this line closely approximates the graph of ' f ' for values of x near xo, if follows that

$$
\begin{equation*}
\mathrm{f}(\mathrm{x}) ? \mathrm{f}\left(\mathrm{x}_{0}\right)+\mathrm{f}^{\prime}\left(\mathrm{x}_{0}\right)\left(\mathrm{x}-\mathrm{x}_{0}\right) \tag{1}
\end{equation*}
$$

Provided x is close to $\mathrm{x}_{0}$. We call (1) the local linear approximation of ' f ' at $\mathrm{x}_{0}$. Furthermore, it can be shown that (1) is actually the best linear approximation of ' $f$ ' near $x_{0}$ in the sense that any other linear function will fail to give as good as approximation to $f$ for values of x very close to $\mathrm{x}_{0}$. An alternative version of this formula can be obtained by letting ? $\mathrm{x}=\mathrm{x}-\mathrm{x}_{0}$ in which case (1) can be expressed as

$$
\mathrm{f}\left(\mathrm{x}_{0}+? \mathrm{x}\right) ? \mathrm{f}\left(\mathrm{x}_{0}\right)+\mathrm{f}^{\prime}\left(\mathrm{x}_{0}\right) ? \mathrm{x}
$$

In practice we use the following method for local linear approximation

1) We are given some value to calculate say $f(x)$, and the calculation is difficult.
2) We see that there is a nearby point $x 0$, where the calculation of both $f(x 0)$ and $f\left(x_{0}\right)$ ' is relatively easy.
3) We approximate $f(x)$ by $\left.f\left(x_{0}\right)+f\left(x_{0}\right)\right)^{\prime}\left(x-x_{0}\right)$

Example: Find the local linear approximation of $f(x)=$ ?x at $x 0=1$
Solution: Given $f(x)=? x, f^{\prime}(x)=1 /(2$ ? $x)$
The local linear approximation of ? $x$ at $x 0=1$ is
?x ? ? $1+1 / 2(x-1)=1 / 2(x+1)$

## Approximating Changes - Differentials



The second use ofderivative is to approximate small changes in a function. Start
at point $x$ and move a small distance given by the independent variable ?x. Then define ?y to be the corresponding change in the value of $y=f(x)$. We can see that $? \mathrm{y}=\mathrm{f}(\mathrm{x}+$ ? x$)-\mathrm{f}(\mathrm{x})$

$$
\begin{aligned}
y+? y & =f(x+? x) \\
? y & =f(x+? x)-y \\
& =f(x+? x)-f(x)
\end{aligned}
$$

Let $d x$ be an arbitrary variable and define dy to be $f^{\prime}(x) d x$. Then the ratio of $d y$ to $d x$ is the derivative $d y / d x$. $d x$ and dy are called the differentials.
Example: Find the differential dy when $y=x^{18}$ when $x=1$
Solution: $y=x{ }^{18}$
$d y=18 x^{17}{ }_{d x}$
$=18 \mathrm{dx}$ since $1^{17}=1$

## Error Propagation in Applications

In applications, small errors invariably occur in measured quantities. When these quantities are used in computations, those errors are propagated in turn to the computed quantities. For example, suppose that in an application the variables x and y are related by a function $\mathrm{y}=\mathrm{f}(\mathrm{x})$. If $\mathrm{x}_{\mathrm{a}}$ is the actual value of x , and it is measured to be $\mathrm{x}_{0}$, then we define the difference $\mathrm{dx}=\mathrm{x}_{0}-\mathrm{x}_{\mathrm{a}}$ to be the error in measurement of $x$. If the error is positive, the measured value is larger than the actual value and if the error is negative the measured value is smaller than the actual value. Since $y$ is determined from $x$ by the function $y=f(x)$, the true value of $y$ is $f\left(x_{a}\right.$ ) and the value of $y$ computed from the measured value of $x$ is $f\left(x_{0}\right)$. The propagated error in the computed value of $y$ is defined to be $f\left(x_{0}\right)-f\left(x_{a}\right)$. If the propagated error is positive, the calculated value of $y$ will be too large and if this error is negative, the calculated value of y will be too small.
The local linear approximation (1) becomes

$$
\begin{aligned}
\mathrm{f}(\mathrm{xa}) & ? \mathrm{f}(\mathrm{x} 0)+\mathrm{f}^{\prime}(\mathrm{x} 0)(\mathrm{xa}-\mathrm{x} 0) \\
& ? \mathrm{f}(\mathrm{x} 0)-\mathrm{f}^{\prime}(\mathrm{x} 0)(\mathrm{x} 0-\mathrm{xa}) \\
& ? \mathrm{f}\left(\mathrm{x}_{0}\right)-\mathrm{f}^{\prime}\left(\mathrm{x}_{0}\right) \mathrm{dx}
\end{aligned}
$$

Example: Suppose that the side of a square is measured with a ruler to be 10 inches with a measurement error of at most $\pm 1 / 32$ of an inch. Use a differential to estimate the error in the computed area of the square.
Solution: The side of a square $x$ and the area of the square $y$ are related by the equation $y=x^{2}$
Since $d y=2 x d x$ and if $x=10$ then $d y=20 d x$
To say that the measurement error is at most $\pm 1 / 32$ of an inch means that the measurement error $d x=x_{0}-x_{a}$ satisfies the
inequalities $\quad-1 / 32$ ?dx?1/32. Multiplying each term by 20 yields the equivalent inequalities
20(-1/32)? dy ?20(1/32)
-5/8 ?dy ?5/8
Since we are using the differential dy to approximate the propagated error, we estimate this propagated error to be between $-5 / 8$ and $5 / 8$ of a square inch. Hence we estimate the propagated error to be at most $\pm 5 / 8$ of a square inch.

## Relative error and Percentage error

The ratio of the error in some measured or calculated quantity to the true value of the quantity is called the relative error of the measurement or calculation. When expressed as a percentage, the relative error is called the percentage error. For example, suppose that the side of a square is measured to be 10 inches, but the actual length of the side is 9.98 inches, the relative error in this measurement is 0.02/9.98?0.002004008.

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## Reference Links:

- http://en.wikipedia.org/wiki/Linear_approximation
- http://www.buzzle.com/articles/relative-error.html
- http://en.wikipedia.org/wiki/Derivative
- http://en.wikipedia.org/wiki/Variable_(mathematics)

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