INVERTIBLE MATRICES

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What are Invertible matrices?

$$A^{-1} = \frac{1}{|A|}\begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{32} & A_{33} \end{bmatrix}$$

If A is a square matrix of order m, and if there exists another square matrix B of the same order m such that AB = BA = I, then B is called the inverse matrix of A and is denoted by A^{-1}. In this case we say A is invertible.

Important Remarks:

- Inverse of a square matrix, if it exists, is unique.
- If A and B are invertible matrices of the same order, then $(AB)^{-1} = B^{-1}A^{-1}$

Inverse of a matrix by elementary operations

There are six operations (transformations) on a matrix, three of which are due to rows and three due to columns, which are known as elementary operations or transformations.

i) The interchange of any two rows or two columns. Symbolically the interchange of i^{th} and j^{th} rows is denoted by $R_i \longleftrightarrow R_j$ and the interchange of i^{th} and j^{th} column is denoted by $C_i \leftrightarrow C_j$

ii) The multiplication of the elements of any row or column by a non zero number. Symbolically the multiplication of each element of the i^{th} row by k, where k?0 is denoted by $R_i \longleftrightarrow kR_i$. The corresponding column operation is denoted by $C_i \longleftrightarrow kC_i$

iii) The addition to the elements of any row or column, the corresponding elements of any other row or column multiplied by any non zero number. Symbolically, the addition to the elements of ith row, the corresponding elements of jth row multiplied by k is denoted by $R_i \leftrightarrow R_i+kR_j$ The corresponding column operation is denoted by $C_i \leftrightarrow C_i+kC_j$

Example: Obtain the inverse of the following matrix using elementary operation

$$A = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{pmatrix}$$

Solution: We know A=IA

$$\begin{pmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} A$$

$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 3 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} A \qquad R_1 \longleftrightarrow R_2$$

$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & -5 & -8 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & -3 & 1 \end{pmatrix} A \qquad R_3 \longrightarrow R_1 - 3R_1$$

$$\begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & -5 & -8 \end{pmatrix} = \begin{pmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & -3 & 1 \end{pmatrix} A \qquad R_1 \longrightarrow R_1 - 2R_2$$

$$\begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{pmatrix} = \begin{pmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ 5 & -3 & 1 \end{pmatrix} A \qquad R_3 \longrightarrow R_3 + 5R_2$$

$$\begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ 5/2 & -3/2 & ½ \end{pmatrix} A \qquad R_3 \longrightarrow ½\, R_3$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} ½ & -1/2 & ½ \\ 1 & 0 & 0 \\ 5/2 & -3/2 & ½ \end{pmatrix} A \qquad R_1 \longrightarrow R_1 + R_2$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} ½ & -1/2 & ½ \\ -4 & 3 & -1 \\ 5/2 & -3/2 & ½ \end{pmatrix} A \qquad R_2 \longrightarrow R_2 - 2R_3$$

$$\text{Hence } A^{-1} = \begin{pmatrix} ½ & -1/2 & ½ \\ -4 & 3 & -1 \\ 5/2 & -3/2 & ½ \end{pmatrix}$$

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Reference Links:

http://www.britannica.com/EBchecked/topic/561660/square-matrix

http://en.wikipedia.org/wiki/Invertible_matrix

http://www.purplemath.com/modules/mtrxrows.htm

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