

Point of intersection and concurrency of lines

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Point of Intersection of Two Lines

Let the equations of two lines be

$$a_1x + b_1y + c_1 = 0 \text{ -- (i)}$$

$$a_2x + b_2y + c_2 = 0 \text{ -- (ii)}$$

Suppose these two lines [intersect](#) at a point P(x₁, y₁). Then, (x₁, y₁) satisfies each of the given equations.

Therefore, $a_1x_1 + b_1y_1 + c_1 = 0$ and $a_2x_1 + b_2y_1 + c_2 = 0$

Solving these two by [cross multiplication](#), we get

$$\begin{aligned} \frac{x_1}{b_1c_2 - b_2c_1} &= \frac{y_1}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1} \\ x_1 &= \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1} \\ y_1 &= \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1} \end{aligned}$$

Hence, the coordinates of the point of intersection of (i) and (ii) are:

$$\left(\frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}, \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1} \right)$$

Important Remark

1. To find the coordinates of the point of intersection of two non – parallel lines, we solve the given equations simultaneously and the values of x and y so obtained determine the coordinates of the point of intersection.

Condition of Concurrency of three lines

Three lines are said to be concurrent if they pass through a common point i.e. they meet at a point.

Thus, if three lines are [concurrent](#) the point of intersection of two lines lies on the third line. Let

$$a_1x + b_1y + c_1 = 0 \text{ -- (i)}$$

$$a_2x + b_2y + c_2 = 0 \text{ -- (ii)}$$

$$a_3x + b_3y + c_3 = 0 \text{ -- (iii)}$$

be three concurrent lines.

Then the point of intersection of (i) and (ii) must lie on the third. The coordinates of the point of intersection of (i) and (ii) are

$$\left(\frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}, \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1} \right)$$

This point must lie on (iii)

Therefore, $a_3 \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1} + b_3 \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1} + c_3 = 0$

$$a_3(b_1c_2 - b_2c_1) + b_3(c_1a_2 - c_2a_1) + c_3(a_1b_2 - a_2b_1) = 0$$

$$a_1 \ b_1 \ c_1$$

$$a_2 \ b_2 \ c_2 = 0$$

$a_3 \ b_3 \ c_3$

This is the required condition of concurrency of three lines.

Another condition of concurrency of three lines

Three lines:

$$L1 \quad a_1x + b_1y + c_1 = 0$$

$$L2 \quad a_2x + b_2y + c_2 = 0$$

$$L3 \quad a_3x + b_3y + c_3 = 0$$

are concurrent iff there exist constants $\lambda_1, \lambda_2, \lambda_3$ not all zero such that

$$\lambda_1 L1 + \lambda_2 L2 + \lambda_3 L3 = 0$$

$$\lambda_1 (a_1x + b_1y + c_1) + \lambda_2 (a_2x + b_2y + c_2) + \lambda_3 (a_3x + b_3y + c_3) = 0$$

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Reference Links :

- <http://en.wikipedia.org/wiki/Intersection>
- <http://en.wikipedia.org/wiki/Cross-multiplication>
- http://en.wikipedia.org/wiki/Concurrent_lines

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