Algebraic Methods of Solving a Pair of Linear Equations

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Pair of linear equations

A pair of the linear equation is in the following form-

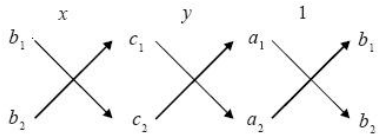
$$a_1x + b_1y + c_1 = 0$$

and $$a_2x + b_2y + c_2 = 0$$

The most commonly used algebraic methods of solving a pair oflinear equations in two variables are –

- Substitution method
- Elimination method
- Cross Multiplication method

Cross Multiplication Method



Let a1x + b1y + c1=0

a2x + b2y + c2=0

be a system ofsimultaneous linear equations in two variables x and y such that a1/a2 ? b1/b2

i.e. a1b2 – a2b1 ? 0. Then the system has a unique solution given by

$$x = \frac{(b_1c_2 - b_2c_1)}{(a_1b_2 - a_2b_1)}$$

$$\text{and } y = \frac{(c_1a_2 - c_2a_1)}{(a_1b_2 - a_2b_1)}$$

Here are the steps which we follow while solving a pair of linear equations by cross multiplication method:

Step I – Obtain the two equations.

Step II – Shift all terms on LHS in the two equations to introduce zeros on RHS i.e., write the two equations in the following form:

a1x + b1y + c1=0

a2x + b2y + c2=0

Step III – In the above system of equations there are three columns viz.

column containing x i.e. $\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$

column containing y i.e. $\begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$

and column containing constant terms i.e. $\begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$

To obtain the solution, write x, -y and 1 separated by equality signs as shown below:

$$\frac{x}{\begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}} = \frac{-y}{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}} = \frac{1}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

In the denominator of x leave column containing x and write remaining two columns in the same order, in the denominator of –y leave column containing y and write the remaining two columns. Similarly, in the denominator of one write columns containing x and y.

Step IV – To obtain the denominators of x, -y and 1, cross multiply the numbers and subtract the product. Applying this, we get

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

Step V – Obtain the value of x by equating first and third expression in step IV. The value of y is obtained by equating second and third expression in step IV.

To get a more clear idea, let's explain with an example:

Example:Solve the following system of equations by using the method of cross multiplication:

x + y = 7

5x + 12y = 7

The given system of equations is

x + y – 7 = 0

5x +12y – 7 = 0

By cross – multiplication, we get

$$\frac{x}{1 \times -7 - 12 \times -7} = \frac{-y}{1 \times -7 - 5 \times -7} = \frac{1}{1 \times 1 - 5 \times 12}$$

$$\frac{x}{1 \times (-7) - 12 \times (-7)} = \frac{-y}{1 \times (-7) - 5 \times (-7)} = \frac{1}{1 \times 12 - 5 \times 1}$$

$$\frac{x}{-7 + 84} = \frac{-y}{-7 + 35} = \frac{1}{12 - 5}$$

$$\frac{x}{77} = \frac{-y}{28} = \frac{1}{7}$$

x = 77/7 and y = -28/7

x = 11 and y = -4

Try these questions now:

1. Solve the following system of equations by using the method of cross multiplication:

2x + 3y = 17

3x - 2y = 6

(Answer: x = 4 and y = 3)

2. Solve the following system of equations by using the method of cross multiplication:

2x - y = 3

4x + y = 3

(Answer: x = 1 and y = -1)

Now try it yourself! Should you still need any help, click here to schedule live online session with e Tutor!

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Reference Links :

http://en.wikipedia.org/wiki/Linear_equation

http://en.wikipedia.org/wiki/Cross-multiplication

http://en.wikipedia.org/wiki/Simultaneous_equations

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