## HYPERBOLA

Created: Monday, 12 September 2011 10:01 | Published: Monday, 12 September 2011 10:01 | Written by Super User | Print

## Introduction to Hyperbolas



A hyperbola is an open curve with two branches, the intersection of a plane with both halves of a double cone.


The two fixed points F1 and F2 are called the foci of the hyperbola. The midpoint of the line segment joining the foci is called the centre of the hyperbola. The line through the foci is called the transverse axis and the line through the centre and perpendicular to the transverse axis is called the conjugate axis. The points at which the hyperbola intersects the transverse axis are called the vertices of the hyperbola.

be 2 c , and the distance between the two vertices A and B be 2 a . With the assumed information, we can define b as
$\mathrm{b}=? \mathrm{c}^{2}-\mathrm{a}^{2}$

Also 2 b is the length of the conjugate axis.

By taking the point P at A and B in the above figure, we have
$\mathrm{BF}_{1}-\mathrm{BF}_{2}=\mathrm{AF} 2-\mathrm{AF} 1($ By definition $)$
$\mathrm{BA}+\mathrm{AF}_{1}-\mathrm{BF}_{2}=\mathrm{AB}+\mathrm{BF}_{2}-\mathrm{AF}_{1}$
i.e., $\mathrm{AF}_{1}=\mathrm{BF}_{2}$

So that, $\mathrm{BF}_{1}-\mathrm{BF}_{2}=\mathrm{BA}+\mathrm{AF}_{1}-\mathrm{BF}_{2}=\mathrm{BA}=2 \mathrm{a}$


## Equation of Hyperbola

Above diagrams represent two possible orientations of hyperbola.
Now, we will derive theequation for the hyperbola with foci on the x - axis. Consider the following diagram:


Let F and F be the foci and O be the mid-point of the line segment F
${ }_{1} \mathrm{~F}_{2}$. Let O be the origin and the line through O via $\mathrm{F}_{2}$ be the positive $\mathrm{x}-\mathrm{axis}$ and that through $\mathrm{F}_{1}$ as the negative $\mathrm{x}-$ axis. Let the
coordinates of F 1 be $(-\mathrm{c}, 0)$ and F 2 be $(\mathrm{c}, 0)$.
Let $P(x, y)$ be any point on the hyperbola such that the difference between the farther and closer point be 2 a
? $\mathrm{PF}_{1}-\mathrm{PF}_{2}=2 \mathrm{a}-(\mathrm{i})$
Applying distance formula in the above equation:
$?(x+c)^{2}+y^{2}-?(x-c)^{2}+y^{2}=2 a$
$?(x+c)^{2}+y^{2}=2 a+?(x-c)^{2}+y^{2}$
On squaring both sides, we get:
$(x+c)^{2}+y^{2}=4 a^{2}+4 a ?(x-c)^{2}+y^{2}+(x-c)^{2}+y^{2}$
On further simplifying:
$c x / a-a=?(x-c)^{2}+y^{2}$
On squaring both sides again and simplifying further:
$x^{2} / a^{2}-y^{2} /\left(c^{2}-a^{2}\right)=1$
$x^{2} / a^{2}-y^{2} / b^{2}=1\left(\right.$ Since $\left.c^{2}-a^{2}=b^{2}\right)$
Hence, any point on the hyperbola satisfies
$x^{2} / a^{2}-y^{2} / b^{2}=1$

To summarise, here are the observations from the standard equations of parabola:

1. Hyperbola is symmetric with respect to both the axes, since if $(x, y)$ is a point on the hyperbola, then $(-x, y),(x,-y)$ and ( $-x,-$ y) are also points on the hyperbola.
2. The foci are always on the transverse axis. It is the positive term whose denominator gives the transverse axis.

## Eccentricity

For the hyperbola, $\mathbf{x}^{2} / \mathbf{a}^{\mathbf{2}}-\mathbf{y}^{\mathbf{2} / \mathrm{b}^{2}=\mathbf{1}}$
We have, $\mathrm{b}^{2}=\mathrm{a}^{2}\left(\mathrm{e}^{2}-1\right)$
$e^{2}=\left(a^{2}+b^{2}\right) / a^{2}$
$e^{2}=1+b^{2} / a^{2}$
$\mathrm{e}=? 1+\mathrm{b}^{2} / \mathrm{a}^{2}$
$\mathrm{e}=? 1+(2 \mathrm{~b})^{2} /(2 \mathrm{a})^{2}$
$\mathrm{e}=? 1+(\text { Conjugate axis })^{2} /(\text { Transverse axis })^{2}$

## Latus Rectum

Latus rectum of hyperbola is a line segment perpendicular to the transverse axis through any of the foci and whose end points lie on the hyperbola.

Length of latus rectum in hyperbola is $2 \mathrm{~b}^{2} / \mathrm{a}$

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## Reference Links:

- http://en.wikipedia.org/wiki/Hyperbola
- http://www.answers.com/topic/conjugate-axis
- http://www.answers.com/topic/transverse-axis
- http://en.wikipedia.org/wiki/Vertex_(geometry)
- http://en.wikipedia.org/wiki/Hyperbola\#In_Cartesian_coordinates

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