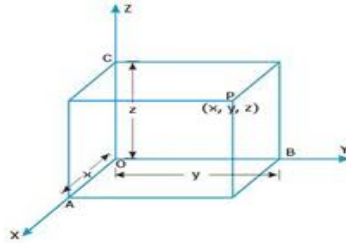


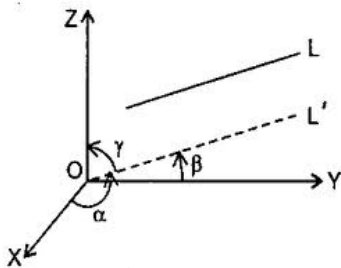
Direction Cosines and Direction Ratios of a Line

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Introduction



We have already learned the basic concepts of vectors. In this topic we will use the concepts of vector algebra to the three dimensional geometry. In the [three dimensional geometry](#), we deal with direction cosines, direction ratios, equations of line in space, equation of plane in space etc.



Direction Cosines

If a directed line L' passing through the origin makes angles α , β and γ with x , y and z axes respectively then cosine of these angles namely,

$\cos \alpha$, $\cos \beta$ and $\cos \gamma$ are called [direction cosines](#) of the directed line L' .

Usually the direction cosines are denoted by l , m and n

$l = \cos \alpha$, $m = \cos \beta$ and $n = \cos \gamma$

Relation between the direction cosines of a line

If l , m and n are the direction cosines of a line then $l^2 + m^2 + n^2 = 1$

Also, $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$

Direction cosines of a line passing through two points

Let $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ be two points on a line L , then

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Direction cosines of the line L is given by, $\frac{x_2 - x_1}{PQ}, \frac{y_2 - y_1}{PQ}, \frac{z_2 - z_1}{PQ}$

Direction Ratios of a line

Any three numbers which are proportional to the direction cosines of a line are called [direction ratios](#) of the line. If l, m and n are direction cosines and a, b and c are direction ratios of a line then $a = \lambda l, b = \lambda m$ and $c = \lambda n$.

$$\frac{l}{a} = \frac{m}{b} = \frac{n}{c} = \lambda$$

It can also be written as

If $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ are any two points the direction ratios of PQ is given by $\langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle$

Direction cosines of x, y and z-axis

X-axis makes angles $0^\circ, 90^\circ$ and 90° with itself, so the direction cosines are $\cos 0^\circ, \cos 90^\circ$ and $\cos 90^\circ = \langle 1, 0, 0 \rangle$

Y-axis makes angles $90^\circ, 0^\circ$ and 90° with itself, so the direction cosines are $\cos 90^\circ, \cos 0^\circ$ and $\cos 90^\circ = \langle 0, 1, 0 \rangle$

Z-axis makes angles $90^\circ, 90^\circ$ and 0° with itself, so the direction cosines are $\cos 90^\circ, \cos 90^\circ$ and $\cos 0^\circ = \langle 0, 0, 1 \rangle$

Condition for collinearity

If a_1, b_1, c_1 and a_2, b_2, c_2 are the direction cosines of line joining two points then the points are said to be collinear

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

if

Example: Find the direction cosines of a line which makes equal angles with the coordinate axes.

Solution: Given $\alpha = \beta = \gamma$, so $\cos \alpha = \cos \beta = \cos \gamma$

$$l = m = n$$

$$l^2 + m^2 + n^2 = 1$$

$$l^2 + l^2 + l^2 = 1$$

$$3l^2 = 1$$

$$l^2 = 1/3$$

$$l = \pm 1/\sqrt{3}$$

$$l = m = n = \pm 1/\sqrt{3}$$

Hence direction cosines are $\langle \pm 1/\sqrt{3}, \pm 1/\sqrt{3}, \pm 1/\sqrt{3} \rangle$

Now try it yourself! Should you still need any help, [click here](#) to schedule live online session with e Tutor!

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Reference Links:-

- http://en.wikipedia.org/wiki/Direction_cosine
- <http://www.solitaryroad.com/c400.html>
- http://en.wikipedia.org/wiki/Three-dimensional_space

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