## Various forms of a Plane

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## Plane - Introduction



A plane can be determined uniquely if anyone of the following is known:
(i) The normal to the plane and its distance from origin is given.
(ii) It passes through a point and is perpendicular to a given direction.
(iii) It passes through three given non collinear points.

## Equation of plane in normal form

Vector Form: If $\overline{\mathrm{r}}$ is the position vector of a point P in the plane, d is the perpendicular distance from origin and ? is the unit normal to the plane then its vector equation is given by

$$
\overline{\mathrm{r}} . ?=\mathrm{d}
$$

Cartesian Form: If $\mathrm{P}(\mathrm{x}, \mathrm{y}, \mathrm{z})$ is a point in the plane, d is the perpendicular distance from origin and $<1, \mathrm{~m}, \mathrm{n}>$ are the direction cosines of ?, then the Cartesian form of the plane is given by

$$
1 \mathrm{x}+\mathrm{my}+\mathrm{nz}=\mathrm{d}
$$

Note: If $\langle\mathrm{a}, \mathrm{b}, \mathrm{c}>$ are the direction ratios of the normal to the plane then the equation is $\mathrm{ax}+\mathrm{by}+\mathrm{cz}=\mathrm{d}$

## Equation of a plane perpendicular to a given vector and passing through a given point

Vector Form: If $\bar{a}$ is the position vector of a given point and $\overline{\mathrm{N}}$ is the perpendicular vector then its equation is given by

$$
(\overline{\mathrm{r}}-\overline{\mathrm{a}}) \cdot \overline{\mathrm{N}}=0
$$

Cartesian Form: If $\mathrm{A}(\mathrm{x} 1, \mathrm{y} 1, \mathrm{z} 1)$ is the given point and $\mathrm{P}(\mathrm{x}, \mathrm{y}, \mathrm{z})$ is a general point in the plane and $\mathrm{A}, \mathrm{B}$ and C are the direction ratios of $\overline{\mathrm{N}}$ then the Cartesian equation is given by

$$
\mathrm{A}\left(\mathrm{x}-\mathrm{x}_{1}\right)+\mathrm{B}\left(\mathrm{y}-\mathrm{y}_{1}\right)+\mathrm{C}\left(\mathrm{z}-\mathrm{z}_{1}\right)=0
$$

## Equation of a plane passing through three non collinear points

Vector Form: If $\bar{a}, \bar{b}$ and $\bar{c}$ are the position vectors of three points and $\overline{\mathrm{r}}$ be any point in the plane, then the equation of the plane passing through three given points is

$$
(\overline{\mathrm{r}}-\overline{\mathrm{a}}) \cdot[(\overline{\mathrm{b}}-\overline{\mathrm{a}}) X(\overline{\mathrm{c}}-\overline{\mathrm{a}})]=0
$$

Cartesian Form: If $\left(\mathrm{x}_{1}, \mathrm{y} 1, \mathrm{z} 1\right),(\mathrm{x} 2, \mathrm{y} 2, \mathrm{z} 2)$ and $(\mathrm{x} 3, \mathrm{y} 3, \mathrm{z} 3)$ are the three given points then equation of the plane is
$\left|\begin{array}{ccc}\mathbf{x}-\mathbf{x}_{1} & \mathbf{y}-\mathbf{y}_{1} & \mathbf{z}-\mathbf{z}_{1} \\ \mathbf{x}_{2}-\mathbf{x}_{1} & \mathbf{y}_{2}-\mathbf{y}_{1} & z_{2}-z_{1} \\ \mathbf{x}_{3}-\mathbf{x}_{1} & \mathbf{y}_{3}-\mathbf{y}_{1} & z_{3}-\mathbf{z}_{1}\end{array}\right|=\mathbf{0}$

## Intercept Form of a plane



If the plane makes intercepts $\mathrm{a}, \mathrm{b}$ and c on $\mathrm{x}, \mathrm{y}$ and z axes respectively
then its equation in intercept form is given by


-     -         - 

Here coordinates of $\mathrm{A}, \mathrm{B}$ and C are $\mathrm{A}(\mathrm{a}, 0,0), \mathrm{B}(0, \mathrm{~b}, 0)$ and $\mathrm{C}(0,0, \mathrm{c})$ respectively.

## Intersection of two planes

Vector Form: If $\overline{\mathrm{r}} . \bar{n}_{1}=\mathrm{d}_{1}$ and $\overline{\mathrm{r}} . \overline{\mathrm{n}}_{2}=\mathrm{d}_{2}$ are the vector equation of two planes then equation of the plane passing through the intersection of these two planes is given by

$$
\overline{\mathrm{r}} .\left(\overline{\mathrm{n}}_{1}+? \overline{\mathrm{n}}_{2}\right)=\mathrm{d}_{1}+? \mathrm{~d}_{2}
$$

Cartesian Form: If $A_{1} x+B_{1} y+C_{1} z=d_{1}$ and $A_{2} x+B_{2} y+C_{2} z=d_{2}$ are the equations of two planes in the Cartesian form then the equation of the plane passing through the intersection of the given planes is

$$
\left(\mathrm{A}_{1} \mathrm{x}+\mathrm{B}_{1} \mathrm{y}+\mathrm{C}_{1} \mathrm{z}-\mathrm{d}_{1}\right)+?\left(\mathrm{~A}_{2} \mathrm{x}+\mathrm{B}_{2} \mathrm{y}+\mathrm{C}_{2} \mathrm{z}-\mathrm{d}_{2}\right)=0
$$

In general, if $P_{1}$ and $P_{2}$ are the equations of two planes then the equation of the plane passing through the intersection of $P_{1}$ and $P_{2}$ is given by

$$
\mathrm{P}_{1}+? \mathrm{P}_{2}=0
$$

Example: Find the equation of the plane through the intersection of the planes $x+y+z-6=0$ and $2 x+3 y+4 z+5=0$ ant the point $(1,1,1)$

Solution: Equation of the plane passing through $x+y+z-6=0$ and $2 x+3 y+4 z+5=0$ is given by

$$
\begin{equation*}
(x+y+z-6)+?(2 x+3 y+4 z+5)=0 \tag{1}
\end{equation*}
$$

Passes through ( $1,1,1$ )
$(1+1+1-6)+?(2+3+4+5)=0$
?=3/14

Substitute the value of? in (1), so that the equation is
$(x+y+z-6)+3 / 14(2 x+3 y+4 z+5)=0$
$20 x+23 y+26 z-69=0$, which is the required equation.

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## Reference links:

- http://en.wikipedia.org/wiki/Plane_\(geometry\)
- http://en.wikipedia.org/wiki/Surface_normal
- http://www.cs.fit.edu/~wds/classes/cse5255/thesis/planeEqn/planeEqn.html
- http://www.wikidoc.org/index.php/Plane \%28mathematics\%29
- http://mathworld.wolfram.com/Plane.html

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