Various forms of a Plane

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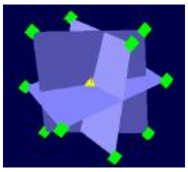
Plane - Introduction



A plane can be determined uniquely if anyone of the following is known:

(i) The normal to the plane and its distance from origin is given.

(ii) It passes through a point and is perpendicular to a given direction.

(iii) It passes through three given non collinear points.

Equation of plane in normal form

Vector Form: If $\bar{r}$ is the position vector of a point P in the plane, d is the perpendicular distance from origin and ? is the unit normal to the plane then its vector equation is given by

$$\bar{r}. ? = d$$

Cartesian Form: If P(x, y, z) is a point in the plane, d is the perpendicular distance from origin and <l, m, n> are the direction cosines of ?, then the Cartesian form of the plane is given by

$$lx + my + nz = d$$

Note: If <a, b, c> are the direction ratios of the normal to the plane then the equation is $ax + by + cz = d$

Equation of a plane perpendicular to a given vector and passing through a given point

Vector Form: If $\bar{a}$ is the position vector of a given point and $\bar{N}$ is the perpendicular vector then its equation is given by

$$(\bar{r} - \bar{a}).\bar{N} = 0$$

Cartesian Form: If A(x1, y1, z1) is the given point and P(x, y, z) is a general point in the plane and A, B and C are the direction ratios of $\bar{N}$ then the Cartesian equation is given by

$$A(x-x_1)+B(y-y_1)+C(z-z_1)=0$$

Equation of a plane passing through three non collinear points

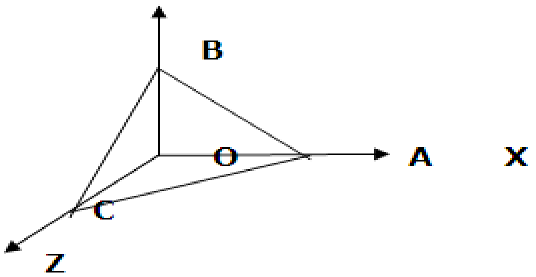
Vector Form: If $\bar{a}$, $\bar{b}$ and $\bar{c}$ are the position vectors of three points and $\bar{r}$ be any point in the plane, then the equation of the plane passing through three given points is

$$(\bar{r} - \bar{a}).[(\bar{b} - \bar{a}) \text{ X } (\bar{c} - \bar{a})]=0$$

Cartesian Form: If (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3) are the three given points then equation of the plane is

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

Intercept Form of a plane



If the plane makes intercepts a, b and c on x, y and z axes respectively then its equation in intercept form is given by

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

Here coordinates of A, B and C are A(a,0,0), B(0,b,0) and C(0,0,c) respectively.

Intersection of two planes

Vector Form: If $\bar{r}.\,\bar{n}_1=d_1$ and $\bar{r}.\,\bar{n}_2=d_2$ are the vector equation of two planes then equation of the plane passing through the intersection of these two planes is given by

$$\bar{r}.(\bar{n}_1+?\,\bar{n}_2)=d_1+?d_2$$

Cartesian Form: If $A_1x+B_1y+C_1z=d_1$ and $A_2x+B_2y+C_2z=d_2$ are the equations of two planes in the Cartesian form then the equation of the plane passing through the intersection of the given planes is

$$(A_1x+B_1y+C_1z-d_1)+?(A_2x+B_2y+C_2z-d_2)=0$$

In general, if P_1 and P_2 are the equations of two planes then the equation of the plane passing through the intersection of P_1 and P_2 is given by

$$P_1+?P_2=0$$

Example: Find the equation of the plane through the intersection of the planes x+y+z-6=0 and 2x+3y+4z+5=0 ant the point (1, 1, 1)

Solution: Equation of the plane passing through x+y+z-6=0 and 2x+3y+4z+5=0 is given by

(x+y+z-6) + ? (2x+3y+4z+5)=0 ……………(1)

Passes through (1, 1, 1)
(1+1+1-6) + ? (2+3+4+5) = 0
?=3/14

Substitute the value of ? in (1), so that the equation is
(x+y+z-6)+3/14(2x+3y+4z+5) = 0
20x+23y+26z-69=0, which is the required equation.

Now try it yourself! Should you still need any help, click here to schedule live online session with e Tutor!

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Reference links:

- http://en.wikipedia.org/wiki/Plane_%28geometry%29
- http://en.wikipedia.org/wiki/Surface_normal
- http://www.cs.fit.edu/~wds/classes/cse5255/thesis/planeEqn/planeEqn.html
- http://www.wikidoc.org/index.php/Plane_%28mathematics%29
- http://mathworld.wolfram.com/Plane.html

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