Vector and Cartesian Equations of a Line

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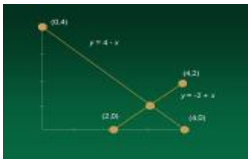
Equation of a line in space

We have studied equation of lines in previous classes. Now we will learn the vector and Cartesian equation of a line in space.

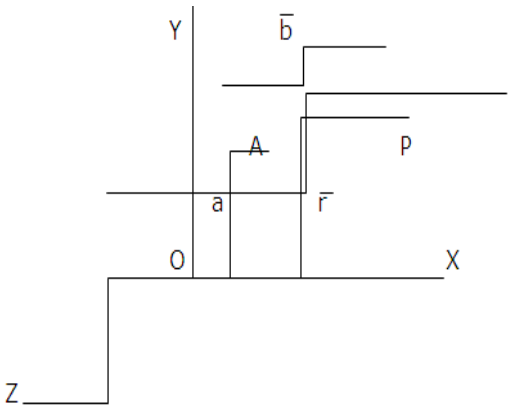


A line is uniquely determined if,

i) It passes through a given point and has given direction
ii) It passes through two given points.

Equation of a line through a given point and parallel to a given vector

Let $\bar{a}$ be given position vector of the given point and $\bar{b}$ be the given vector, then its equation is given by $\bar{r}=\bar{a}+?\bar{b}$



Vector Equation

In the above figure $\overline{AP}$ is parallel to $\bar{b}$, so $\overline{AP}=?\bar{b}$ ……(1)

$\overline{AP}= \overline{OP} - \overline{OA}$

$= \bar{r} - \bar{a}$

(1) becomes, $\bar{r} - \bar{a} = ?\bar{b}$

$\bar{r}= \bar{a}+?\bar{b}$, which is the vector equation.

Hence[vector equation](#) of a line passing through a point with position vector a and parallel to a given vector b is given by $\bar{r}=\bar{a}+?\bar{b}$

Cartesian Equation

Let the coordinates of the given point be $A(x_1, y_1, z_1)$ and the direction ratios of the parallel vector be <a, b, c>. Let P(x, y,z) be any point (General point) on the line.

The Cartesian equation is given by $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$

Example: Find the vector and Cartesian equations of the line through the point (5,3,-5) and which is parallel to the vector 4î-7?+3k

Solution: We have $\bar{a}$=5î+3?-5k and $\bar{b}$= 4î-7?+3k, so

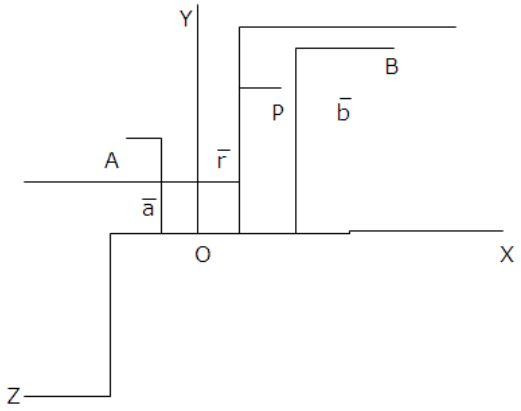
Vector equation is $\bar{r}= \bar{a} +?\bar{b}$

= (5î+3?-5k) +? (4î-7?+3k)

Cartesian Equation is $\frac{x-5}{4} = \frac{y-3}{-7} = \frac{z-(-5)}{3}$

Equation of a line passing through two points

Let$\bar{a}$ and$\bar{b}$ be the position vectors of two points that are lying on a a given line then their equation is given by$\bar{r}=\bar{a}+?(\bar{b}-\bar{a})$



Vector Equation

Let $\bar{a}$ and $\bar{b}$ be the position vectors of the points lying on the line and $\bar{r}$ be the position of any point (general point).

We know $\overline{AP}$ and $\overline{AB}$ are collinear vectors, therefore P will lie on the line if and only if $\overline{AP} = ? \overline{AB}$

$\bar{r}-\bar{a} = ? (\bar{b}-\bar{a})$

$\bar{r} = \bar{a} + ? (\bar{b}-\bar{a})$, which is the vector equation.

Cartesian Equation

Let A(x1, y1, z1) and B(x2, y2, z2) be two point in the line and P(x, y, z) be a general point on the line, the Cartesian Equation is given by

$$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$$

Example: Find the Vector and Cartesian equation of the line joining the points (-1,3,2) and (3,0,1)

Solution: Here $\bar{a}$= -î+3?+2k and $\bar{b}$=3î+0?+k

Vector equation is $\bar{r}$= (-î+3j+2k)+?(4î-3?-k)

Cartesian equation is $\frac{x-(-1)}{4} = \frac{y-3}{-3} = \frac{z-2}{-1}$

Now try it yourself! Should you still need any help,click here to schedule live online session with e Tutor!

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Reference links:

- http://en.wikipedia.org/wiki/Position_%28vector%29
- http://www.revisesmart.co.uk/maths/core-4/vector-equation-of-a- line.html
- http://en.wikiversity.org/wiki/Vectors

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