Angle between two planes

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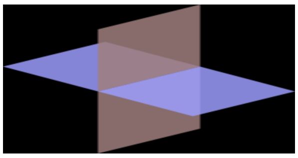
Angle between two planes - Introduction



The angle between two planes is defined as the angle between their normals If ? is the angle between two planes, then so is 180 - ?. We shall take the acute angle as the angle between two planes

Vector Form: If $\bar{r}.\bar{n}_1=d_1$ and $\bar{r}.\bar{n}_2=d_2$ are the equation of two planes then angle between them is given by the equation

$$\cos\theta = \left| \frac{\bar{n}_1.\bar{n}_2}{|\bar{n}_1||\bar{n}_2|} \right|$$

$$\theta = \cos^{-1}\left(\frac{\bar{n}_1.\bar{n}_2}{|\bar{n}_1||\bar{n}_2|} \right)$$

Cartesian Form: If $A_1x + B_1y + C_1z + D_1=0$ and $A_2x + B_2y + C_2z + D_2=0$ are the Cartesian equations of two planes and ? is the angle between them then

$$\cos\theta = \left| \frac{A_1A_2 + B_1B_2 + C_1C_2}{\sqrt{A_1^2 + B_1^2 + C_1^2}\sqrt{A_2^2 + B_2^2 + C_2^2}} \right|$$

Condition for parallelism and perpendicularity

1. If the planes are parallel then $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

2. If the planes are perpendicular then $A_1A_2 + B_1B_2 + C_1C_2=0$

Coplanarity of Two Lines

Vector Form: If $\bar{r} = \bar{a}_1 + ?\bar{b}_1$ and $\bar{r} = \bar{a}_2 + ?\bar{b}_2$ are the equations of two lines then they are said to be coplanar if $(\bar{a}_2 - \bar{a}_1).(\bar{b}_1 \times \bar{b}_2) = 0$

Cartesian Form: If $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ are two points with the direction ratios of parallel vectors $<a_1, b_1, c_1>$ and $<a_2, b_2, c_2>$, then the lines are said to be coplanar if

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

Distance of a point from a plane

Vector Form: If the equation of the plane is in the form $\bar{r}.\bar{N}=d$, where $\bar{N}$ is normal to the plane, then the perpendicular distance is

$$\frac{|\bar{a}.\bar{N}-d|}{|\bar{N}|}$$

The length of perpendicular from origin O to the plane $\bar{r}.\bar{N}=d$ is $|d|/|\bar{N}|$

Cartesian Form: If $P(x_1, y_1, z_1)$ be the given point with position vector $\bar{a}$ and $Ax + By + Cz=D$ be the equation of the plane then the perpendicular distance from P to the plane is given by $d = \left| \dfrac{Ax_1+By_1+Cz_1-D}{\sqrt{A^2+B^2+C^2}} \right|$

Angle between a Line and a Plane

If $\bar{r}=\bar{a}+?\bar{b}$ be the equation of the line and $\bar{r}.\bar{n}=d$ be the equation of the plane the angle between them is given by

$$\sin\Phi = \left| \frac{\bar{b}.\bar{n}}{|\bar{b}||\bar{n}|} \right|$$

$$\Phi = \sin^{-1}\left(\frac{\bar{b}.\bar{n}}{|\bar{b}||\bar{n}|} \right)$$

Example: Find the distance of a point (2, 5, -3) from the plane $6x - 3y + 2z - 4=0$

Solution: Distance $d = \dfrac{|6x2-3x5+2x-3-4|}{\sqrt{36+9+4}}$

$$= \frac{|12-15-6-4|}{\sqrt{49}}$$

= 13/7

Now try it yourself! Should you still need any help, click here to schedule live online session with e Tutor!

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Links:

- http://en.wikipedia.org/wiki/Angle#Angles_between_curves
- http://schools-wikipedia.org/wp/p/Plane_%2528mathematics%2529.htm
- http://www.netcomuk.co.uk/~jenolive/vect
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