INVERSE TRIGONOMETRIC FUNCTIONS

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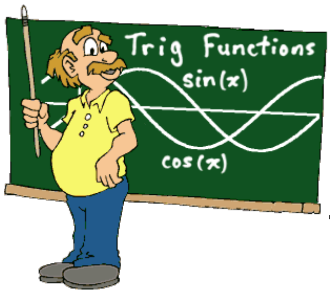
What are inverse trigonometric functions?



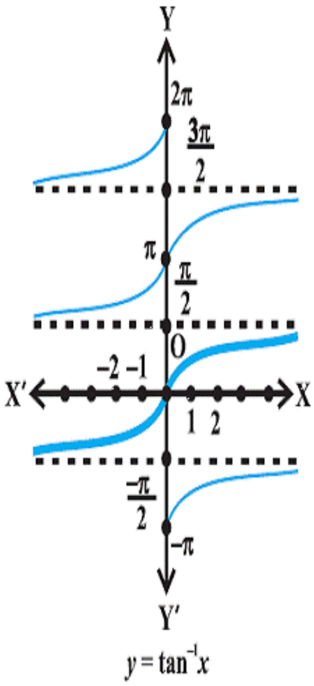
The inverse trigonometric functions are the inverse functions of the trigonometric functions, though they do not meet the official definition for inverse functions as their ranges are subsets of the domains of the original functions.

Domain and Range of Inverse Trigonometric Functions

Function Name	Domain	Range
1. Arc Sine	-1 ? x ? 1	- ?2 ? y ? ?2
2. Arc Cosine	-1 ? x ? 1	0 ? y ? ?
3. Arc Tangent	All real numbers	- ?2 ? y ? ?2
4. Arc Cosecant	x ? ?1 or 1 ? x	- ?2 ? y < 0 or 0 < y ? ?2
5. Arc Secant	x ? ?1 or 1 ? x	0 ? y < ?2 or ?2 < y ? ?
6. Arc Cotangent	All real numbers	0 < y < ?

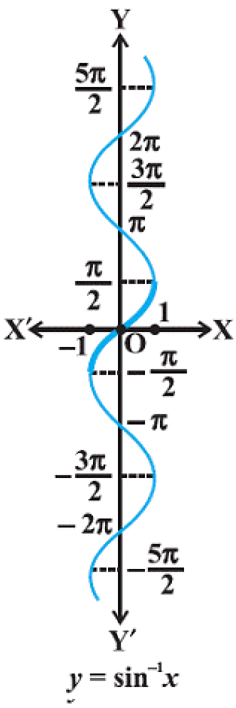
Graphs of Inverse Trigonometric Functions

As discussed above, the domain and range of all inverse trigonometric functions, we shall now represent each of them on graph.

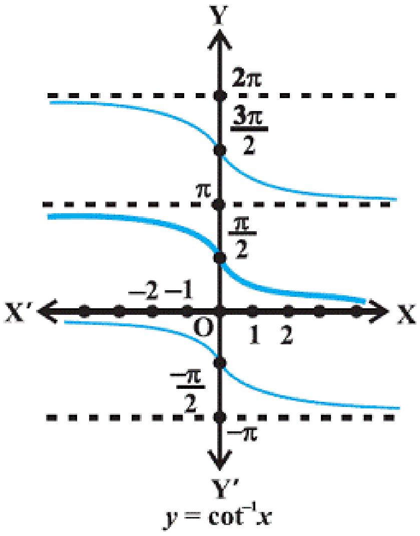


Sin-1x **Cos-1x** **Tan-1x**

Cosec-1x **Sec-1x**



Cot-1x



Properties of Inverse Trigonometric Functions

1. (i) $\text{Sin}^{-1}1x$= $\text{Cosec}^{-1}x$, x ? 1 or x ? -1

Proof: Let Cosec-1x = y, that is x = Cosec y

1x = Sin y

$\text{Sin}^{-1}1x$ = y

$\text{Sin}^{-1}1x$ = Cosec-1x

(ii) $\text{Cos}^{-1}1x$ = $\text{Sec}^{-1}x$, x ? 1 or x ? -1

Proof: Let Sec-1x = y, that is x = Sec y

1x = Cos y

$\text{Cos}^{-1}1x$ = y

$\text{Cos}^{-1}1x$ = Sec-1x

(iii) $\text{Tan}^{-1}1x$ = $\text{Cot}^{-1}x$, x > 0

Proof: Let Cot-1x = y that is x = Cot y

1x = Tan y

$\text{Tan}^{-1}1x$ = y

$\text{Tan}^{-1}1x$ = $\text{Cot}^{-1}x$

2. (i) $\text{Sin}^{-1}(-x) = -\text{Sin}^{-1}x$, x ? [-1, 1]

Proof: Let $\text{Sin}^{-1}(-x) = y$

$-x = \text{Sin } y$

$x = -\text{Sin } y$

$x = \text{Sin }(-y)$

$\text{Sin}^{-1}x = -y = -\text{Sin}^{-1}(-x)$

$\text{Sin}^{-1}(-x) = -\text{Sin}^{-1}x$

(ii) $\text{Tan}^{-1}(–x) = – \text{Tan}^{-1} x$, x ? R

Proof: Let $\text{Tan}^{-1}(-x) = y$

$-x = \text{Tan } y$

$x = -\text{Tan } y$

$x = \text{Tan }(-y)$

$\text{Tan}^{-1}x = -y = -\text{Tan}^{-1}(-x)$

$\text{Tan}^{-1}(-x) = -\text{Tan}^{-1}x$

(iii) $\text{Cosec}^{-1}(–x) = – \text{Cosec}^{-1} x$, | x | ? 1

Proof: Let $\text{Cosec}^{-1}(-x) = y$

$-x = \text{Cosec } y$

$x = -\text{Cosec } y$

$x = \text{Cosec }(-y)$

$\text{Cosec}^{-1}x = -y = -\text{Cosec}^{-1}(-x)$

$\text{Cosec}^{-1}(-x) = -\text{Cosec}^{-1}x$

3. (i) $\text{Cos}^{-1}(–x) =$? $– \text{Cos}^{-1} x$, x ? [– 1, 1]

Proof: Let $\text{Cos}^{-1}(–x) = y$ i.e., $– x = \text{Cos } y$ so that $x = – \text{Cos } y = \text{Cos }$(? – y)

$\text{Cos}^{-1} x =$? – y = ? $– \text{Cos}^{-1}(–x)$

Hence $\text{Cos}^{-1}(–x) =$? $– \text{Cos}^{-1} x$

(ii) $\text{Sec}^{-1}(–x) =$? $– \text{Sec}^{-1} x$, | x | ? 1

Proof: Let $\text{Sec}^{-1}(–x) = y$ i.e., $– x = \text{Sec } y$ so that $x = – \text{Sec } y = \text{Sec }$(? – y)

$\text{Sec}^{-1} x =$? – y = ? $– \text{Sec}^{-1}(–x)$

Hence $Sec^{-1}(-x) = ? - Sec^{-1} x$

(iii) $Cot^{-1}(-x) = ? - Cot^{-1} x, x ? R$

Proof: Let $Cot^{-1}(-x) = y$ i.e., $-x = Cot\ y$ so that $x = -Cot\ y = Cot\ (? - y)$

$Cot^{-1} x = ? - y = ? - Cot^{-1}(-x)$

Hence $Cot^{-1}(-x) = ? - Cot^{-1} x$

4. (i) $Sin^{-1} x + Cos^{-1} x = ?2$, $x ? [-1, 1]$

Let $Sin^{-1} x = y$. Then $x = Sin\ y = Cos\ (?2 - y)$

$Cos^{-1} x = ?2 - y$

$Cos^{-1} x = ?2 - Sin^{-1} x$

Hence, $Sin^{-1} x + Cos^{-1} x = ?2$

(ii) $Tan^{-1} x + Cot^{-1} x = ?2$, $x ? R$

Let $Tan^{-1} x = y$. Then $x = Tan\ y = Cot\ (?2 - y)$

$Cot^{-1} x = ?2 - y$

$Cot^{-1} x = ?2 - Tan^{-1} x$

$Tan^{-1} x + Cot^{-1} x = ?2$

(iii) $Cosec^{-1} x + Sec^{-1} x = ?2$,$| x | ? 1$

Let $Cosec^{-1} x = y$. Then $x = Cosec\ y = Sec\ (?2 - y)$

$Sec^{-1} x = ?2 - y$

$Sec^{-1} x = ?2 - Cosec^{-1} x$

$Sec^{-1} x + Cosec^{-1} x = ?2$

5. $Tan\ x^{-1} + Tan^{-1} y = Tan^{-1} \dfrac{x + y}{1 - xy}$, $xy < 1$

Proof: Let $Tan^{-1}x = ?$ and $Tan^{-1}y = ?$

$x = Tan\ ?$ and $y = Tan\ ?$

Now $Tan\ (? + ?) = \dfrac{Tan\ ? + Tan\ ?}{1 - Tan\ ?\ Tan\ ?}$

$$\text{Tan}(? + ?) = \frac{x + y}{1 - xy}$$

$$? + ? = \text{Tan}^{-1}\frac{x + y}{1 - xy}$$

$$\text{Tan}\,x^{-1} + \text{Tan}^{-1}y = \text{Tan}^{-1}\frac{x + y}{1 - xy} \text{ – (I)}$$

In the above equation (I) replace y by (-y), we get another result:

$$\text{Tan}\,x^{-1} - \text{Tan}^{-1}y = \text{Tan}^{-1}\frac{x - y}{1 + xy},\ xy > -1$$

If we replace y by x in equation (I) we get:

$$2\,\text{Tan}^{-1}x = \text{Tan}^{-1}\frac{2x}{1 - x^2},\ ?x?< 1$$

6. (i) $$2\,\text{Tan}^{-1}x = \text{Sin}^{-1}\frac{2x}{1 + x^2},\ ?x??\ 1$$

(ii) $$2\,\text{Tan}^{-1}x = \text{Cos}^{-1}\frac{1 - x^2}{1 + x^2},\ x\ ?\ 0$$

(iii) $$2\,\text{Tan}^{-1}x = \text{Tan}^{-1}\frac{2x}{1 - x^2},\ -1 < x < 1$$

Now try it yourself! Should you still need any help, click here to schedule live online session with e Tutor!

Reference Links:

- http://www.answers.com/topic/inverse-function
- http://www.answers.com/topic/trigonometric-function
- http://www.answers.com/topic/range-mathematics
- http://www.answers.com/topic/subset-1
- http://www.answers.com/topic/domain-mathematics

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