TRIGONOMETRIC FUNCTIONS OF MULTIPLE AND SUBMULTIPLE ANGLES

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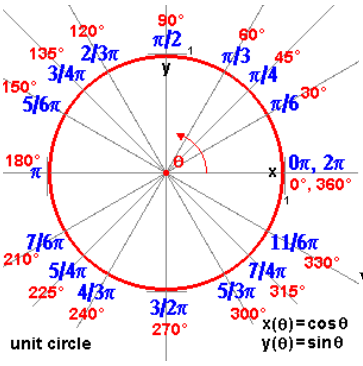
Trigonometric Functions Introduction



We have learnt about trigonometric functions of the sum and difference of two angles using some basic results, under this section using the trigonometric ratios of the sum and difference of two angles we will define the trigonometric ratios of multiple and submultiple angles. We must be familiar with the following identities before starting this topic:

1. Cos (A + B) = Cos A Cos B – Sin A Sin B
2. Cos (A – B) = Cos A Cos B + Sin A Sin B
3. Sin (A + B) = Sin A Cos B + Cos A Sin B
4. Sin (A – B) = Sin A Cos B - Cos A Sin B
5. Tan (A + B) = Tan A +Tan B1-Tan A Tan B
6. Tan (A – B) = Tan A - Tan B1+Tan A Tan B

Trigonometric Ratios of Angle 2A in terms of Angle A

- Sin 2A = 2 Sin A Cos A

As we know, Sin (A + B) = Sin A Cos B + Cos A Sin B

Replacing B by A in the above equation
Sin (A + A) = Sin A Cos A + Cos A Sin A
Sin 2A = 2 Sin A Cos A

- Cos 2A = Cos2 A – Sin2 A
  $= 2 \text{Cos}^2 A - 1$
  $= 1 - 2 \text{Sin}^2 A$

As we know, Cos (A + B) = Cos A Cos B – Sin A Sin B

Replacing B by A in the above equation
Cos (A + A) = Cos A Cos A – Sin A Sin A
$\text{Cos } 2A = \text{Cos}^2 A - \text{Sin}^2 A$

- $\text{Cos } 2A = \text{Cos}^2 A - \text{Sin}^2 A$

$\text{Cos } 2A = (1 - \text{Sin}^2 A - \text{Sin}^2 A)$
$\text{Cos } 2A = 1 - 2 \text{Sin}^2 A$

- $\text{Cos } 2A = \text{Cos}^2 A - \text{Sin}^2 A$

$\text{Cos } 2A = \text{Cos}^2 A - (1 - \text{Cos}^2 A)$
$\text{Cos } 2A = 2 \text{Cos}^2 A - 1$

- Tan 2A = 2 Tan A
  $1 - \text{Tan}^2 A$

As we know, Tan (A + B) = Tan A +Tan B1-Tan A Tan B

Replacing B by A in the above equation
Tan (A + A) = Tan A +Tan A1-Tan A Tan A
Tan 2A = 2 Tan A
$1 - \text{Tan}^2 A$

Trigonometric Ratios of Angle 3A in terms of Angle A

- Sin 3A = 3 Sin A – 4 Sin3 A

As we know, Sin (A + B) = Sin A Cos B + Cos A Sin B

Replacing B by 2A in the above equation
Sin (A + 2A) = Sin A Cos 2A + Cos A Sin 2A
$\text{Sin } 3A = \text{Sin } A (1 - 2 \text{Sin}^2 A) + \text{Cos } A (2 \text{Sin } A \text{Cos } A)$

$\text{Sin } 3A = \text{Sin } A - 2 \text{Sin}^3 A + 2 \text{Sin } A \text{Cos}^2 A$

$\text{Sin } 3A = \text{Sin } A – 2 \text{ Sin}^3 A + 2 \text{ Sin } A (1 – \text{Sin}^2 A)$

$\text{Sin } 3A = \text{Sin } A – 2 \text{ Sin}^3 A + 2 \text{ Sin } A – 2 \text{ Sin}^2 A$

$\text{Sin } 3A = 3 \text{ Sin } A – 4 \text{ Sin}^3 A$

- Cos 3A= 4 Cos3 A – 3 Cos A

As we know, Cos (A + B) = Cos A Cos B – Sin A Sin B

Replacing B by 2A in the above equation
$\text{Cos } (A + 2A) = \text{Cos } A \text{ Cos } 2A – \text{Sin } A \text{ Sin } 2A$
$\text{Cos } 3A = \text{Cos } A (2 \text{ Cos}^2 A – 1) – \text{Sin } A (2 \text{ Sin } A \text{ Cos } A)$
$\text{Cos } 3A = 2 \text{ Cos}^3 A – \text{Cos } A – 2 \text{ Sin}^2 A \text{ Cos } A$
$\text{Cos } 3A = 2 \text{ Cos}^3 A – \text{Cos } A – 2 (1 – \text{Cos}^2 A) \text{ Cos } A$
$\text{Cos } 3A = 2 \text{ Cos}^3 A – \text{Cos } A – 2 \text{ Cos } A + 2 \text{ Cos}^3 A$
$\text{Cos } 3A = 4 \text{ Cos}^3 A – 3 \text{ Cos } A$

- Tan 3A = $3 \text{ Tan } A – \text{Tan}^3 A$
  $1 – 3 \text{ Tan}^2 A$

As we know, Tan (A + B) = Tan A +Tan B1-Tan A Tan B

Replacing B by 2A in the above equation
Tan (A + 2A) = Tan A +Tan 2A1-Tan A Tan 2A

- Tan 3A = Tan A + 2 Tan A

  $1 – \text{Tan}^2 A$
  1 – Tan A 2 Tan A
  $1 – \text{Tan}^2 A$

On Solving we get:
$\text{Tan } 3A = 3 \text{ Tan } A – \text{Tan}^3 A$
$1 – 3 \text{ Tan}^2 A$

Now try it yourself! Should you still need any help,click here to schedule live online session with e Tutor!

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Reference Links:

- http://en.wikipedia.org/wiki/Trigonometric_functions
- http://en.wikipedia.org/wiki/List_of_trigonometric_identities
- http://www.purplemath.com/modules/idents.htm

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