TRIGONOMETRIC FUNCTIONS OF SUM AND DIFFERENCE OF TWO ANGLES

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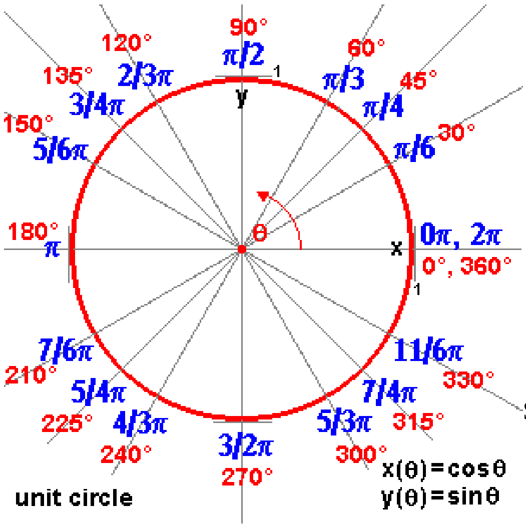
Trigonometric Functions of Sum and Differences Introduction

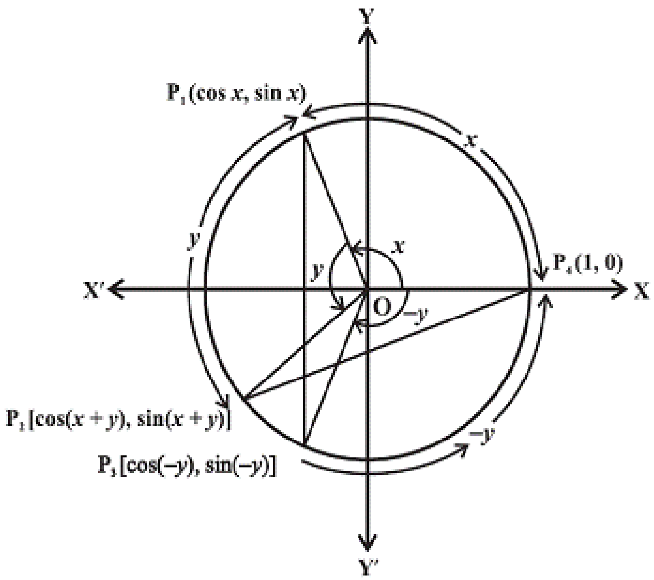


So far we have learnt basic trigonometric identities, in this section we will learn about trigonometric functions of the sum and difference of two angles using some basic results, following are the results which we must know before starting this topic:

1. Sin (-x) = - Sin x
2. Cos (-x) = Cos x
3. Sin (?2 – x) = Cos x
4. Cos (?2 – x) = Sin x
5. Sin (?2 + x) = Cos x
6. Cos (?2 + x) = - Sin x
7. Cos (? – x) = - Cos x
8. Sin (? – x) = Sin x
9. Cos (? + x) = - Cos x
10. Sin (? + x) = - Sin x
11. Cos (2? – x) = Cos x
12. Sin (2? - x) = - Sin x

Basic Proof

Consider theunit circle with centre at the origin.



Let x be the angle P_4OP_1 and y be the [angle]() P_1OP_2. Then (x + y) is the angle P_4OP_2. Also let (– y) be the angle P_4OP_3.

Therefore, P_1, P_2, P_3 and P_4 will have the [coordinates]():

P_1 (Cos x, Sin x),
P_2 [Cos (x + y), Sin (x + y)],
P_3 [Cos (– y), Sin (– y)] and
P_4 (1, 0)

Consider the triangles P_1OP_3 and P_2OP_4. They are congruent

Therefore, P_1P_3 and P_2P_4 are equal. By using [distance formula](), we get
$P_1P_3^2$
$2 = [\text{Cos } x - \text{Cos } (-y)]^2 + [\text{Sin } x - \text{Sin}(-y)]^2$
$= (\text{Cos } x - \text{Cos } y)^2 + (\text{Sin } x + \text{Sin } y)^2$
$= \text{Cos}^2 x + \text{Cos}^2 y - 2 \text{ Cos } x \text{ Cos } y + \text{Sin}^2 x + \text{Sin}^2 y + 2\text{Sin } x \text{ Sin } y$
$= 2 - 2 (\text{Cos } x \text{ Cos } y - \text{Sin } x \text{ Sin } y)$

Also, $P_2P_4^2$
$2 = [1 - \text{Cos } (x + y)]^2 + [0 - \text{Sin } (x + y)]^2$
$= 1 - 2\text{Cos } (x + y) + \text{Cos}^2 (x + y) + \text{Sin}^2 (x + y)$
$= 2 - 2 \text{ Cos } (x + y)$

Since $P_1P_3 = P_2P_4$, we have $P_1P_3^2 = P_2P_4^2$

Therefore, 2 –2 (Cos x Cos y – Sin x Sin y) = 2 – 2 Cos (x + y).

So, from the above discussion we get:
Cos (x + y) = Cos x Cos y – Sin x Sin y – (I)

Cosine: Sum and Difference of Angles

Sum of Angles: Cos (x + y)

From (I) we get:
Cos (x + y) = Cos x Cos y – Sin x Sin y

Difference of Angles: Cos (x – y)

Replacing y by (-y) in (I), we get:
Cos (x + y) = Cos x Cos y – Sin x Sin y
Cos (x + (-y)) = Cos x Cos (-y) – Sin x Sin (-y)
Cos (x – y) = Cos x Cos y + Sin x Sin y (From 1 and 2)

Sine: Sum and Difference of Angles

We will use Cos (?2 – x) = Sin x to find out sum and difference of angles

Sum of Angles: Sin (x + y)

Sin (x + y) = Cos (?2 – (x + y))
Sin (x + y) = Cos ((?2 – x) – y)
= Cos (?2 – x) Cos y + Sin (?2 – x) Sin y (Using (I))
= Sin x Cos y + Cos x Sin y

Difference of Angles: Sin (x – y)

Replacing y by (-y) in sum of angles
Sin (x + (-y)) = Cos (?2 – (x + (-y)))
Sin (x - y) = Cos ((?2 – x) + y)
= Cos (?2 – x) Cos y - Sin (?2 – x) Sin y (Using (I))
= Sin x Cos y - Cos x Sin y

Tangent: Sum and Difference of Angles

We consider that x, y are not an odd multiple of ?2

Sum of Angles: Tan (x + y)

Tan (x + y) = Sin (x+y)Cos (x+y)
Tan (x + y) = Sin x Cos y+Cos x Siny Cos x Cos y-Sin x Sin y

Dividing numerator and denominator by Cos x Cos y, we have:
Tan (x + y) = Tan x +Tan y1-Tan x Tan y

Difference of Angles: Tan (x – y)

Replacing y by (-y) in sum of angles
Tan (x – y) = Tan x- Tan y1+Tan x Tan y

Now try it yourself! Should you still need any help,click here to schedule live online session with e Tutor!

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Reference Links:

- http://en.wikipedia.org/wiki/List_of_trigonometric_identities
- http://www.purplemath.com/modules/idents.htm
- http://en.wikipedia.org/wiki/Unit_circle
- http://en.wikipedia.org/wiki/Angle
- http://www.mathopenref.com/coordpoint.html
- http://en.wikipedia.org/wiki/Analytic_geometry#Distance_and_angle

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