UNIT CIRCLE

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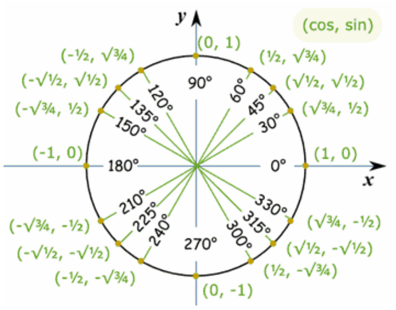
Unit Circle Introduction



The unit circle is the circle with center (0, 0) and radius 1 unit. Consider a circle with center O (0, 0) and radius 1 unit.

Using distance formula, we know that:

$$OP = \sqrt{(x-0)^2 + (y-0)^2}$$

$$= \sqrt{x^2 + y^2}$$

$$x^2 + y^2 = 1$$

Hence, the equation of the unit circle is given by:
$x^2 + y^2 = 1$

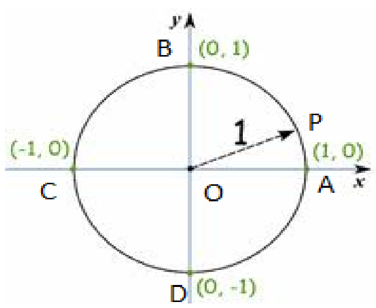
Trigonometric Functions

Consider a unit circle with center at origin of the coordinate axes. Let P (a, b) be any point on the circle with angle AOP = x radian, which means that length of arc AP = x

We define Cos (x) = a and Sin (x) = b

Since ?OMP is a right triangle, we have
$OM^2 + MP^2 = OP^2$
$a^2 + b^2 = 1$ or $Cos^2 x + Sin^2 x = 1$

Since one complete revolution subtends an angle of 2? radian at the center of the circle, ? AOB = ?/2, ? AOC = ? and ? AOD = 3?/2. All angles which are multiples of ?/2 are called quadrantal angles. The coordinates of the points A, B, C, D are respectively (1, 0), (0, 1) (-1, 0) and (0, -1). Therefore, for quadrantal angles, we have



Cos (0) = 1

Sin (0) = 0
Cos (?/2) = 0
Sin (?/2) = 1
Cos (?) = -1
Sin (?) = 0
Cos (3?/2) = 0
Sin (3?/2) = -1
Cos (2?) = 1
Sin (2?) = 0

Now, if we take one complete revolution from the point P, we again come back to same point P. Thus, we also observe that if x increases (decreases) by any integral multiple of 2?, the values of sine and cosine functions do not change. So we can say that Sin (2n?+x) = Sin(x), n ? Z and Cos (2n?+x) = Cos(x), n ? Z

Also Sin (x) = 0 implies x = n?, where n is any integer.
Cos (x) = 0 implies x = (2n+1) ?/2, where n is any integer.

Example: Find the value of Sin (31?/3)

Solution: Sin (31?/3) = $\left(\text{Sin } \frac{10\pi + \pi}{3}\right)$

= Sin (?/3)
= ?3/2

Sign of trigonometric functions

Let P (a, b) be a point of the unit circle with center at the origin such that ? AOP = x. If ? AOQ = -x, then the coordinates of the point Q will be (a, -b).

Therefore, Cos (-x) = Cos (x) and
Sin (-x) = -Sin (x)

Since for every point P (a, b) on the unit circle, -1?a?1 and -1?b?1, we have -1 ? Cos (x) ?1 and -1 ? Sin (x) ? 1 for all x.

The sign of different trigonometric functions is given below:

	I	II	III	IV
Sin(x)	+	+	-	-
Cos(x)	+	-	-	+
Tan(x)	+	-	+	-
Cosec(x)	+	+	-	-
Sec(x)	+	-	-	+
Cot(x)	+	-	+	-

To make the calculations using signs of different trigonometric functions easy memorize the quotation “All Silver Tea Cups” which means that ‘All’ trigonometric functions are positive in 1^{st} quadrant, ’S’ of silver shows that sine and cosecant are positive in 2nd quadrant, ‘T’ of tea shows that tangent and cot are positive in 3rd quadrant and ‘C’ of cups shows that cosine and sec are positive in 4th quadrant.

Now try it yourself! Should you still need any help, click here to schedule live online session with e Tutor!

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Reference Links:

- http://en.wikipedia.org/wiki/Unit_circle
- http://en.wikipedia.org/wiki/Radian
- http://en.wikipedia.org/wiki/Degree_(angle)
- http://en.wikipedia.org/wiki/Trigonometric_functions

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