## UNIT CIRCLE

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## Unit Circle Introduction



The unit circle is the circle with center $(0,0)$ and radius 1 unit. Consider a circle with center $\mathrm{O}(0,0)$ and radius 1 unit.

Using distance formula, we know that:

$$
\begin{aligned}
O P & =\sqrt{(x-0)^{2}+(y-0)^{2}} \\
& =\sqrt{x^{2}+y^{2}} \\
x^{2}+ & y^{2}=1
\end{aligned}
$$

Hence, the equation of the unit circle is given by:
$x^{2}+y^{2}=1$

## Trigonometric Functions

Consider a unit circle with center at origin of the coordinate axes. Let $P(a, b)$ be any point on the circle with angle $A O P=x$ radian, which means that length of $\operatorname{arc} \mathrm{AP}=\mathrm{x}$

We define $\operatorname{Cos}(x)=a$ and $\operatorname{Sin}(x)=b$

Since ?OMP is a right triangle, we have
$\mathrm{OM}^{2}+\mathrm{MP}^{2}=\mathrm{OP}^{2}$
$a^{2}+b^{2}=1$ or $\operatorname{Cos}^{2} x+\operatorname{Sin}^{2} x=1$
Since one complete revolution subtends an angle of 2 ? radian at the center of the circle, ? $\mathrm{AOB}=? / 2, ? \mathrm{AOC}=$ ? and $? \mathrm{AOD}=$ $3 ? / 2$. All angles which are multiples of $? / 2$ are called quadrantal angles. The coordinates of the points $\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}$ are respectively $(1,0),(0,1)(-1,0)$ and $(0,-1)$. Therefore, for quadrantal angles, we have

$\operatorname{Cos}(0)=1$
$\operatorname{Sin}(0)=0$
$\operatorname{Cos}(? / 2)=0$
$\operatorname{Sin}(? / 2)=1$
$\operatorname{Cos}(?)=-1$
$\operatorname{Sin}(?)=0$
$\operatorname{Cos}(3 ? / 2)=0$
$\operatorname{Sin}(3 ? / 2)=-1$
$\operatorname{Cos}(2 ?)=1$
$\operatorname{Sin}(2 ?)=0$
Now, if we take one complete revolution from the point $P$, we again come back to same point $P$. Thus, we also observe that if $x$ increases (decreases) by any integral multiple of 2?, the values of sine and cosine functions do not change. So we can say that Sin $(2 \mathrm{n} ?+\mathrm{x})=\operatorname{Sin}(\mathrm{x}), \mathrm{n} ? \mathrm{Z}$ and $\operatorname{Cos}(2 \mathrm{n} ?+\mathrm{x})=\operatorname{Cos}(\mathrm{x}), \mathrm{n} ? \mathrm{Z}$

Also $\operatorname{Sin}(x)=0$ implies $x=n$ ?, where $n$ is any integer.
$\operatorname{Cos}(\mathrm{x})=0$ implies $\mathrm{x}=(2 \mathrm{n}+1) ? / 2$, where n is any integer.

Example: Find the value of $\operatorname{Sin}(31 ? / 3)$
Solution: $\operatorname{Sin}(31 ? / 3)=\left(\begin{array}{l}\operatorname{Sin} \frac{10 \pi}{3}+\pi \\ \end{array}\right)$

$$
\begin{aligned}
& =\operatorname{Sin}(? / 3) \\
& =? 3 / 2
\end{aligned}
$$

## Sign of trigonometric functions

Let $P(a, b)$ be a point of the unit circle with center at the origin such that $\quad$ AOP $=x$. If ? $A O Q=-x$, then the coordinates of the point Q will be (a, -b).

Therefore, $\operatorname{Cos}(-x)=\operatorname{Cos}(x)$ and
$\operatorname{Sin}(-x)=-\operatorname{Sin}(x)$

Since for every point $\mathrm{P}(\mathrm{a}, \mathrm{b})$ on the unit circle, $-1 ? \mathrm{a} ? 1$ and $-1 ? \mathrm{~b}$ ? 1 , we have $-1 ? \operatorname{Cos}(\mathrm{x}) ? 1$ and $-1 ? \operatorname{Sin}(\mathrm{x}) ? 1$ for all x .

The sign of different trigonometric functions is given below:

|  | I | II | III | IV |
| :--- | :---: | :---: | :---: | :---: |
| $\operatorname{Sin}(x)$ | + | + | - | - |
| $\operatorname{Cos}(x)$ | + | - | - | + |
| $\operatorname{Tan}(x)$ | + | - | + | - |
| $\operatorname{Cosec}(x)$ | + | + | - | - |
| $\operatorname{Sec}(x)$ | + | - | - | + |
| $\operatorname{Cot}(x)$ | + | - | + | - |

To make the calculations using signs of different trigonometric functions easy memorize the quotation "All Silver Tea Cups" which means that 'All' trigonometric functions are positive in $1^{\text {st }}$ quadrant, 'S' of silver shows that sine and cosecant are positive in 2nd quadrant, ' T ' of tea shows that tangent and cot are positive in 3rd quadrant and ' C ' of cups shows that cosine and sec are
positive in 4th quadrant.

Now try it yourself! Should you still need any help, click here to schedule live online session with e Tutor!

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## Reference Links:

- http://en.wikipedia.org/wiki/Unit_circle
- http://en.wikipedia.org/wiki/Radian
- http://en.wikipedia.org/wiki/Degree_(angle)
- http://en.wikipedia.org/wiki/Trigonometric_functions

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