LAWS OF SINE, COSINE AND TANGENT

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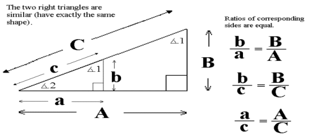
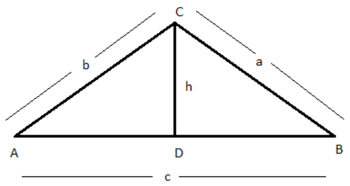
Sine Rule

For any triangle ABC, $\frac{a}{\text{Sin (A)}} = \frac{b}{\text{Sin (B)}} = \frac{c}{\text{Sin (C)}}$



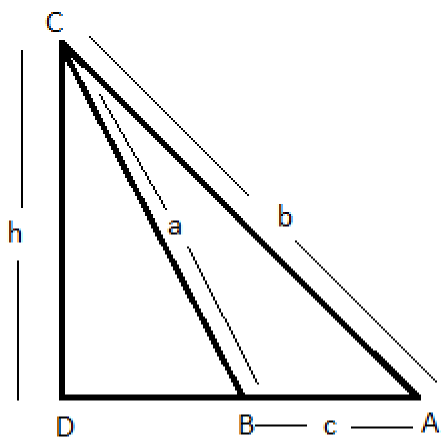
Where a, b and c are the sides opposite to ? A, ? B and ? C respectively is called sine rule. For proving the result we have two cases:



Case I: The perpendicular height is drawn inside the triangle.

Case II: The perpendicular height is drawn outside the triangle.

Let the sides be AB = c, BC = a and AC = b. Let the perpendicular height be CD = h. In case I the perpendicular CD is inside the triangle and in case II the perpendicular is outside the triangle.



In ?ACD, Sin (A) = h/b [Case I]

h = b Sin (A) – (i)

Also, in ?BCD

$\text{Sin}(B) = h/a$ [Case I]

$\text{Sin}(180-B) = h/a$ [Case II]

$\text{Sin}(B) = h/a$

$h = a\,\text{Sin}(B)$ – (ii)

From (i) and (ii), we have

$b\,\text{Sin}(A) = a\,\text{Sin}(B)$

$$\frac{b}{\text{Sin}(B)} = \frac{a}{\text{Sin}(A)}$$ [Rearranging the terms]

Similarly if the perpendicular is drawn from A, we get

$$\frac{b}{\text{Sin}(B)} = \frac{c}{\text{Sin}(C)}$$

Combining both the equations we get

$$\frac{a}{\text{Sin}(A)} = \frac{b}{\text{Sin}(B)} = \frac{c}{\text{Sin}(C)}$$

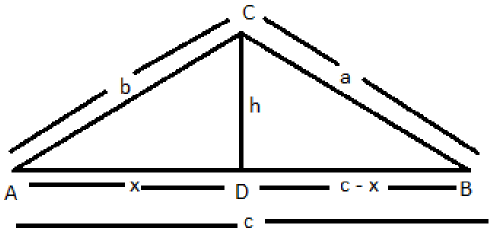
Cosine Rule

For any triangle ABC, $a^2 = b^2 + c^2 - 2bc\,\text{Cos}(A)$, where a, b and c are the sides opposite to ?A, ?B and ?C respectively. The equation above can also be written as

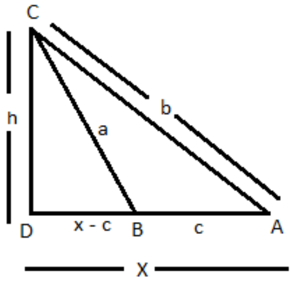
$$\text{Cos}(A) = \frac{b^2 + c^2 - a^2}{2bc}$$

Here also there are two cases:

Case I: Perpendicular height is drawn inside the triangle.



Case II: Perpendicular height is drawn outside the triangle.



In ?ACD, applying Pythagoras' Theorem, we have

$$b^2 = x^2 + h^2 – (i)$$

$$\text{Cos } (A) = x/b$$

$$x = b \text{ Cos } (A) – (ii)$$

In ?BCD, Case I, we have

By Pythagoras' Theorem, $a^2 = h^2 + (c-x)^2$

In Case II, we have, $a^2 = h^2 + (x-c)^2$

In both the case on expanding we get,

$$a^2 = h^2 + c^2 – 2cx + x^2$$

$$a^2 = x^2 + h^2 + c^2 – 2cx$$

$$a^2 = b^2 + c^2 – 2c\,[b\cos(A)] \qquad \text{[from (i) and (ii)]}$$

$$a^2 = b^2 + c^2 – 2bc \text{ Cos}(A)$$

OR

$$\text{Cos } (A) = \frac{b^2 + c^2 - a^2}{2bc}$$

Thus by drawing the perpendiculars differently we get the remaining formulas such as,

$$\text{Cos}(B) = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\text{Cos}(C) = \frac{a^2 + b^2 - c^2}{2ab}$$

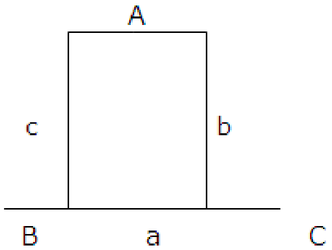
The above mentioned three formulas are known ascosine rule.

Tangent

The law of tangents describes the relationship between the tangent of two angles of a triangle and the lengths of opposite sides. It can be applied to a non-right triangle. The law of tangents is given by,

$$\frac{a - b}{a + b} = \frac{\tan[(A-B)/2]}{\tan[(A+B)/2]}$$

Proof: Consider ?ABC with sides a, b and c which are opposite to the angles A, B and C respectively



To start with the proof, first we must be aware of laws of sines

$$\frac{a}{\mathrm{Sin}\,(A)} = \frac{b}{\mathrm{Sin}\,(B)} = d$$

$$a = d\sin(A) \text{ and } b = d\sin(B)$$

$$\frac{a - b}{a + b} = \frac{d\sin(A) - d\sin(B)}{d\sin(A) + d\sin(B)}$$

$$= \frac{d\,[\sin(A) - \sin(B)}{d\,[\sin(A) + \sin(B)]}$$

$$= \frac{\sin(A) - \sin(B)}{\sin(A) + \sin(B)}$$

$$= \frac{2\cos[(A+B)/2]\,[\sin(A-B)/2]}{2\sin[(A+B)/2]\,[\cos(A-B)/2]}$$

$$= \frac{\tan[(A-B)/2]}{\tan[(A+B)/2]}$$

Hence the law of tangents is proved.

Problems related to these laws

1. In ?ABC ?A=106?, ?B=31? and a = 10cm. Solve ?ABC by calculating ?C and sides ‘b’ and ‘c’ (round your answers to one decimal place).

Solution: In a triangle ? A + ? B + ? C = 180?, ? C= 180? - (106? + 31?)

= 180? - 137?

= 43?

We have a/sin(A) = b/sin(B)

10 = b

$$\frac{}{\text{Sin}(106?)} \quad \frac{}{\text{sin}(31?)}$$

$$b = \frac{10 \times \sin(31?)}{\text{Sin}(106?)}$$

$$= 5.1 \text{ cm}$$

Also we have, $\frac{a}{\sin(A)} = \frac{c}{\sin(C)}$

$$\frac{10}{\sin(106?)} = \frac{c}{\sin(43?)}$$

$$c = \frac{10 \times \sin(43?)}{\text{Sin}(106?)}$$

$$= 7.1\text{cm}$$

2. A triangle has sides equal to 5cm, 10cm and 7cm. Find its angles (Round your answer to one decimal place)

Solution: Let a=10cm, b=7cm and c=5cm

We have $a^2 = b^2 + c^2 - 2bc \text{ Cos } (A)$

$$\text{Cos } (A) = \frac{b^2 + c^2 - a^2}{2bc}$$

$$= \frac{49 + 25 - 100}{2 \times 7 \times 5}$$

$$= -13/35$$

? A= 111.8?

$$\text{Cos}(B) = \frac{a^2 + c^2 - b^2}{2ac}$$

$$= \frac{100 + 25 - 49}{2 \times 10 \times 5}$$

? B = 40.5?

? A + ? B + ? C = 180?

? C = 180 – [111.8? + 40.5?] = 27.7?

Now try it yourself! Should you still need any help, click here to schedule live online session with e Tutor!

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Reference Links:

- http://en.wikipedia.org/wiki/Law_of_cosines
- http://en.wikipedia.org/wiki/Law_of_sines
- http://en.wikipedia.org/wiki/Law_of_tangents
- http://en.wikipedia.org/wiki/Pythagorean_theorem

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