ALTERNATE SERIES TEST

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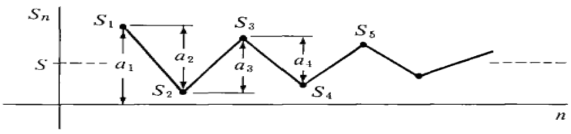
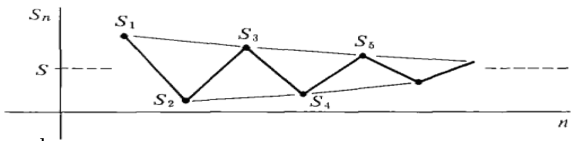
Alternating Series

Series whose terms alternate between positive and negative is called alternating series. They are of special importance. Here are some examples





?$(-1)^{k+1}$ (1/k) = 1 – ½ +1/3 – ¼ + 1/5 - …………

?$(-1)^{k}$ (1/k) = -1 + ½ -1/3 + ¼ -1/5 + ……………

In general, an alternating series has one of the following two forms:

? $(-1)^{k+1}$ a^k = $a_1 - a_2 + a_3 - a_4$ + …………… (1)
?$(-1)^{k}$ a_k = $-a_1 + a_2 - a_3 + a_4$ - …………… (2)

where a_k's are assumed to be positive in both cases.

The following theorem is the key result on convergence of alternating series.

Theorem - (Alternating Series Test): An alternating series of either form (1) or form (2) converges if the following two conditions are satisfied:

(a) a_1 ? a_2 ? a_3 ? …………..? a_k ? …….

(b) $\lim_{k -+?} a_k = 0$

Example: Use the alternating series test to show that the following series converge:

(a) ? $(-1)^{k+1}$ (1/k)

(b) ?$(-1)^{k+1}$ $\frac{k + 3}{k(k+1)}$

Solution: (a) The two conditions in the alternating series test are satisfied since
$a_k = (1/k) > (1/(k+1)) = a_{k+1}$ and $\lim_{k -+?} a_k = \lim_{k-+?} \frac{1}{k} = 0$

(b) The two conditions in the alternating series test are satisfied since

$$\frac{a_{k+1}}{a_k} = \frac{k+4}{(k+1)(k+2)} \cdot \frac{k(k+1)}{k+3} = \frac{k^2 + 4k}{k^2+5k+6} = \frac{k^2 + 4k}{(k^2+4k) + (k+6)} < 1$$

so $a_k > a_{k+1}$ and lim a_k = lim k+3 = lim 1 + 3 = 0

Approximating Sums of Alternating Series

The following theorem is concerned with the error that results when the sum of an alternating series is approximated by a partial sum.

Theorem: If an alternating series satisfies the hypotheses of the alternating series test, and if S is the sum of the series, then:

a) S lies between any two successive partial sums; that is, either s_n? S ?s_{n+1} or s_{n+1} ? S ?s_n depending on which partial sum is larger.

b) If S is approximated by Sn, then the absolute error $|S – s_n|$ satisfies $|S – s_n|$? a_{n+1}

Moreover, the sign of the error $S – s_n$ is the same as that of the coefficient of a_{n+1}.

Absolute Convergence

A series ? $u_k = u_1 + u_2 + u_3 +$ …….. + u_k +………… is said to converge absolutely if the series of absolute values ? $|u_k| = |u_1| + |u_2| + |u_3| +$ …converges and is said to diverge absolutely if the series of absolute values diverges.

Theorem: If the series ?$|u_k| = |u_1| + |u_2| + |u_3| +$ …….+$|u_k|$+…converges, then so does the series ? $u_k = u_1 + u_2 + u_3+$ ……u_k +……..

Example: Show that ? $\frac{\text{Cos (k)}}{k^2}$ converges

Solution: With the help of a calculating utility, you will be able to verify that the signs of the terms in this series vary irregularly. Thus, we will test for absolute convergence. The series of absolute values is ? $|\cos(k)/k^2|$

However, $|\text{Cos}(k)/k^2|$? $1/k^2$

But $1/k^2$ is a convergent p-series (p=2), so the series of absolute values converges by the comparison test. Thus, the given series converges absolutely and hence converges.

Ratio Test for Absolute convergence

Let ?u^k be a series with nonzero terms and suppose that $\rho = \lim_{k \to +\infty} \frac{|u_{k+1}|}{|u_k|}$

a) If ? <1, then the series ? u_k converges absolutely and therefore converges.

b) If ?> 1 or if ? = +?, then the series ?u_k diverges.

c) If ? = 1, no conclusion about convergence or absolute convergence can be drawn from this test.

Example: Use the ratio test for absolute convergence to determine whether the series converges

a) ? $(-1)^k [(2^k)/k!]$

Taking the absolute value of the general term u_k we obtain
$|u_k| = (-1)k[(2^k)/k!] = (2^k)/k!$

$$\text{Thus, } \rho = \lim_{k \to +\infty} \frac{|u_{k+1}|}{|u_k|} = \lim_{k \to +\infty} \frac{2k+1}{(k+1)!} \cdot \frac{k!}{2^k} = \lim_{k \to +\infty} \frac{2}{k+1} = 0 < 1$$

which implies that the series converges absolutely and therefore converges.

b) $?(-1)^k [(2k-1)!/3^k]$

Taking the absolute value of the general term uk we obtain

$$|uk| = \left| (-1)^k \frac{(2k-1)!}{3^k} \right| = \frac{(2k-1)!}{3^k}$$

$$\text{Thus, } \rho = \lim_{k \to +\infty} \frac{|u_{k+1}|}{|u_k|} = \lim_{k \to +\infty} \frac{[2(k+1)-1]!}{3^{k+1}} \cdot \frac{3^k}{(2k-1)!}$$

$$= \frac{1}{3} \lim_{k \to +?} (2k)(2k+1)$$

= +?, which implies that the series diverges.

Now try it yourself! Should you still need any help, click here to schedule live online session with e Tutor!

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Reference Links:

- http://en.wikipedia.org/wiki/Alternating_series
- http://en.wikipedia.org/wiki/Alternating_series_test
- http://en.wikipedia.org/wiki/Absolute_value
- http://en.wikipedia.org/wiki/Ratio_test

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