AREA OF BOUNDED REGIONS

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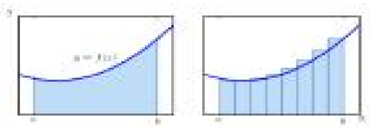
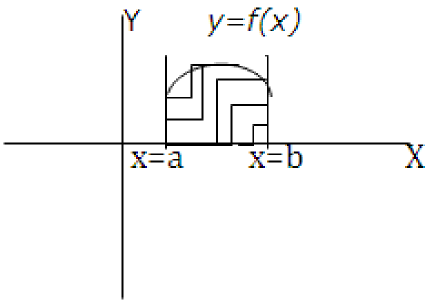
Introduction to Bounded Regions



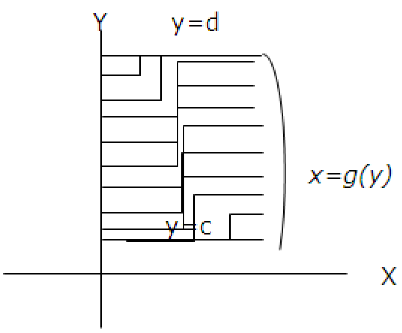
In this article we will study a specific application of integrals to find the area under simple curves, area between lines and arcs of circles, parabolas and ellipses.

Area under simple curves

The area bounded by the curve y = f(x), x-axis and the ordinates x = a and x = b is given by $A={}_{a}?^{b}\ y\ dx={}_{a}?^{b}\ f(x)\ dx$



The area A of the region bounded by the curve x = g(y), y-axis and the lines y = c and y = d is given by
$A = {}_{c}?^{d}\ x\ dy = {}_{c}?^{d}\ g(y)\ dy$

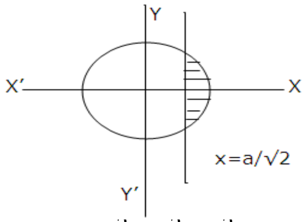


Example: Find the area of the region bounded by the curve $y^2 = 9x$, x = 2, x = 4 and the x-axis in the first quadrant.

Solution: Area = ${}_{2}?^{4}\ y\ dx$
$= {}_{2}?^{4}\ 3?x\ dx$
$= 3\ [x^{3/2} / (3/2)]_{2}^{4}$
$= (16 - 4?2)$ sq. units

Area between a curve and a line

Example: Find the area of the smaller part of the circle $x^2 + y^2 = a^2$ cut off by the line x = a/?2



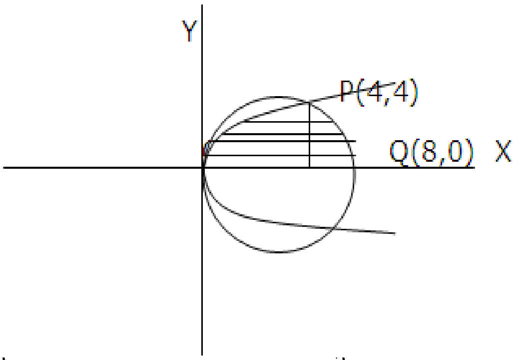
Solution: $x^2 + y^2 = a^2$ ………….(1)
x = a/?2 …………..(2)

Solving (1) and (2) we will get the point of intersection

We have to find the area of shaded region which is given by

A=2*[$_{a/?2}?^{a}$ Area under the curve ($y^2 = a^2 - x^2$) dx]
= 2 $_{a/?2}?^{a}$?(a^2 - x^2) dx
= 2 [x/2 (?a^2 - x^2) – (a^2/2) $\sin^{-1}$ (x/a)]$_{a/?2}^{a}$
= a^2/2 [(?/2) – 1] sq. units

Area between two curves



Example: Find the area lying above x-axis and included between the circle $x + y^2 = 8x$ and inside the parabola $y^2 = 4x$

Solution: The given equation of the circle $x^2 + y^2 = 8x$ can be expressed as $(x - 4)^2 + y^2 = 16$, which is a circle with center (4, 0) and radius 4.

The point of intersection gives x = 0, 4

Hence the curves intersect at O (0, 0) and P (4, 4) above the x-axis.

Required area=2 $_0?^4$?x dx+ $_4?^8$?(4^2 - $(x - 4)^2$) dx
$=2(2/3)\ [x^{3/2}]_0^{\ 4} + [((x-4)/2)\ ?(4^2-(x-2)^2) + (4^2/2)\sin^{-1}(x-2)/2]_4^{\ 8}$
$= 32/3 + [4/2 *0+1/2 *16*\sin^{-1}(1)]$
= (32/3) + 4?
= (4/3) (8 + 3?) sq. units

Now try it yourself! Should you still need any help,click here to schedule live online session with e Tutor!

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Reference Links:

- http://en.wikipedia.org/wiki/Integral
- http://en.wikipedia.org/wiki/Circle#Area_enclosed
- http://wiki.answers.com/Q/Finding_area_of_a_parabola
- http://en.wikipedia.org/wiki/Ellipse#Area

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