

# COMPARISON TEST

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## Introduction

Here we are dealing with a test that is useful in its own right and is also the building block for other important convergence tests. The underlying idea of this test is to use the known [convergence](#) or divergence of a series to deduce the convergence or divergence of another series.

Theorem: Let  $\sum a_k$  and  $\sum b_k$  be series with non-negative terms and suppose that  $a_1 \geq b_1, a_2 \geq b_2, a_3 \geq b_3, \dots, a_k \geq b_k, \dots$

$$\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = L$$

a) If the “bigger series”  $\sum b_k$  converges, then the “smaller series”  $\sum a_k$  also converges.

b) If the “smaller series”  $\sum a_k$  diverges, then the “bigger series”  $\sum b_k$  also diverges.

## Using the Comparison Test

There are two steps required for using the [comparison test](#) to determine whether a series  $\sum u_k$  with positive terms converges:

Step I: Guess at whether the series  $\sum u_k$  converges or diverges.

Step II: Find a series that proves the guess to be correct. That is, if the guess is divergence, we must find a divergent series whose terms are “smaller” than the corresponding terms of  $\sum u_k$ , and if the guess is convergence, we must find a convergent series whose terms are “bigger” than the corresponding terms of  $\sum u_k$ .

In most cases, the series  $\sum u_k$  being considered will have its general term  $u_k$  expressed as a fraction. To help with the guessing process in the first step, we have formulated two principles that are based on the form of the denominator for  $u_k$ . These principles are called informal principles because they are not intended as formal theorems.

**Informal Principle 1:** Constant summands in the denominator of  $u_k$  can be usually be deleted without affecting the convergence or divergence of the series.

**Informal Principle 2:** If a polynomial in ‘k’ appears as a factor in the numerator or denominator of  $u_k$ , all but the leading term in the polynomial can usually be discarded without affecting the convergence or divergence of the series.

## Problems related to comparison test

Use the comparison test to determine whether the following series converge or diverge.

1.  $\sum \frac{1}{k - 1/2}$

2.  $\sum \frac{1}{2k^2 + k}$

1. According to principle 1, we should be able to drop the constant in the denominator without affecting the convergence or

divergence. Thus, the given series is likely to behave like  $\sum 1/\sqrt{k}$  which is a divergent p-series ( $p=1/2$ ). Thus, we will guess that the given series diverges and try to prove this by finding a divergent series that is “smaller” than the given series.

$$\frac{1}{\sqrt{k-1/2}} > \frac{1}{\sqrt{k}} \text{ for } k=1, 2, \dots$$

Thus, we have proved that the given series diverges.

2. According to Principle 2, we should be able to discard all but the leading term in the polynomial without affecting the convergence or divergence. Thus, the given series is likely to behave like  $\sum (1/2k^2) = \frac{1}{2} \sum (1/k^2)$  which converges since it is a constant times a convergent p-series ( $p=2$ ). Thus, we will guess that the given series converges and try to prove this by finding a [convergent series](#) that is “bigger” than the given series.

$$\frac{1}{2k^2 + k} < \frac{1}{2k^2} \text{ for } k=1, 2, 3, \dots$$

Thus, we have proved that the given series converges.

## Limit Comparison Test

In the previous two problems the informal principles provided the guess about convergence or divergence as well as the series needed to apply the comparison test. It is not always easy enough to find the series required for comparison, so we will now consider an alternative to the comparison test that is usually easier to apply.

Let  $\sum a_k$  and  $\sum b_k$  be series with positive terms and suppose that

$$\rho = \lim_{k \rightarrow \infty} \frac{a_k}{b_k}$$

If  $\rho$  is finite and  $\rho > 0$ , then the series both converge or both diverge.

### Problems related to limit comparison test

Use the [limit comparison test](#) to determine whether the following series converge or diverge

1.  $\sum \frac{1}{\sqrt{k-1}}$

2.  $\sum \frac{3k^3 - 2k^2 + 4}{k^7 - k^3 + 2}$

1. To prove that the given series diverges, we will apply the limit comparison test with  $a_k = 1/\sqrt{k-1}$  and  $b_k = 1/\sqrt{k}$

$$\rho = \lim_{k \rightarrow \infty} \frac{a_k}{b_k} = \lim_{k \rightarrow \infty} \frac{\sqrt{k}}{\sqrt{k-1}} = \lim_{k \rightarrow \infty} \frac{1}{1 - (1/\sqrt{k})} = 1$$

We obtain,

Since  $\rho$  is finite and positive, it follows that the given series diverges.

2.  $\rho = \lim_{k \rightarrow \infty} \left( \frac{3k^3 - 2k^2 + 4}{k^7 - k^3 + 2} \right)$

Since  $r$  is finite and nonzero, it follows that the given series converges.

Now try it yourself! Should you still need any help, [click here](#) to schedule live online session with e Tutor!

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## Reference Links:

- [http://en.wikipedia.org/wiki/Convergence\\_\(mathematics\)](http://en.wikipedia.org/wiki/Convergence_(mathematics))
- [http://en.wikipedia.org/wiki/Comparison\\_test](http://en.wikipedia.org/wiki/Comparison_test)
- <http://archives.math.utk.edu/visual.calculus/6/series.9/index.html>
- [http://en.wikipedia.org/wiki/Convergent\\_series](http://en.wikipedia.org/wiki/Convergent_series)

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