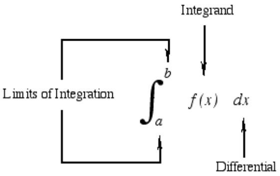
DEFINITE INTEGRALS

Created: Tuesday, 08 November 2011 10:54 | Published: Tuesday, 08 November 2011 10:54 | Written by Super User | Print

Introduction



In this topic, we will learn about definite integral of a function. The definite integral has a unique value. A definite integral is denoted by a?b f(x) dx, where 'a' is called the lower limit of the integral and 'b' is called the upper limit of the integral. The definite integral is introduced either as the limit of a sum or if it has an anti-derivative F in the interval [a, b], then its value is the difference between the values of F at the end points [F(b) – F(a)].

Properties of Definite Integrals

Here are some important properties of definite integrals. Using this property we will be able to solve the definite integrals easily.

1. ${}_a?^b$ f(x) dx = ${}_a?^b$ f(t) dt
2. ${}_a?^b$ f(x) dx = - ${}_b?^a$ f(x) dx. In particular ${}_a?^a$ f(x) dx = 0
3. ${}_a?^b$ f(x) dx = ${}_a?^c$ f(x) dx + ${}_c?^b$ f(x) dx where a < c < b
4. ${}_a?^b$ f(x) dx = ${}_a?^b$ f(a + b - x) dx
5. ${}_0?^a$ f(x) dx = ${}_0?^a$ f(a - x) dx
6. ${}_0?^{2a}$ f(x) dx = ${}_0?^a$ f(x) dx + ${}_0?^a$ f(2a - x) dx
7. ${}_0?^{2a}$ f(x) dx = 2 ${}_0?^a$ f(x) dx, if f(2a - x) = f(x) and
   = 0 if f (2a - x) = - f(x)

8. $_{-a}?^{a}$ f(x) dx = 2 0?a f(x) dx, if 'f' is an even function
$_{-a}?^{a}$ f(x)dx = 0 if 'f' is an odd function
Example: Using properties of integrals evaluate, $_{0}?^{?/2}$ $\cos^{2}$x dx
Solution: Let I = $_{0}?^{?/2}$ $\cos^{2}$x dx – (i)
I = $_{0}?^{?/2}$ $\cos^{2}$ (?/2 – x) dx [Using 5th property]
= $_{0}?^{?/2}$ $\sin^{2}$x dx – (ii)
Adding (i) and (ii)
2I = $_{0}?^{?/2}$ ($\cos^{2}$x + $\sin^{2}$x) dx
= $_{0}?^{?/2}$ 1 dx
= $[x]_{0}^{?/2}$
= ?/2
I = ?/4
Hence $_{0}?^{?/2}$$\cos^{2}$x dx = ?/4

Evaluation of Definite Integrals by Substitution

To evaluate $_{a}?^{b}$ f(x) dx, by substitution, the following steps should be followed.
i) Consider the integral without limits and substitute, y=f(x) or x=g(y) to reduce the given integral to a known form.
ii) Integrated the new integral with respect to the new variable without mentioning the constant of integration.
iii) Re - substitute for the new variable and write the answer in terms of the original variable.
iv) Find the values of answers obtained in (iii) at the given limits of integral and find the difference of the values at the upper and lower limits.

Now try it yourself! Should you still need any help, click here to schedule live online session with e Tutor!

Reference Links:

- http://en.wikipedia.org/wiki/Definite_Integrals
- http://en.wikipedia.org/wiki/Derivative
- http://www.sosmath.com/calculus/integ/integ02/integ02.html
- http://en.wikipedia.org/wiki/Integral