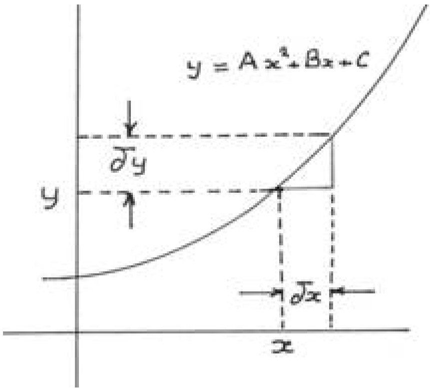
DERIVATIVES OF POLAR AND VECTOR FUNCTION

Created: Tuesday, 08 November 2011 11:53 | Published: Tuesday, 08 November 2011 11:53 | Written by Super User | Print

Differentiation of Vectors



If a vector R varies continuously as a scalar variable t changes, then R is said to be a function of t and is written as R = F(t). Just as in scalar calculus, we define derivative of a vector function R = F(t) as

$$\lim_{\delta t \to 0} \frac{F(t+\delta t) - F(t)}{\delta t}$$ and write it as dR/dt or dF/dt or F'(t).

General rules of differentiation are similar to those of ordinary calculus provided the order of factors in vector products is maintained. Thus, if ?, F, G, H are scalar and vector functions of a scalar variable t, we have

i) $\frac{d}{dt}[F+G-H] = \frac{df}{dt} + \frac{dG}{dt} - \frac{dH}{dt}$

ii) $\frac{d}{dt}[F\Phi] = F\frac{d\Phi}{dt} + \Phi\frac{dF}{dt}$

iii) $\frac{d}{dt}[F.G] = F.\frac{dG}{dt} + \frac{dF}{dt}.G$

i.

iv) $\frac{d}{dt}(F \times G) = F \times \frac{dG}{dt} + \frac{dF}{dt} \times G$

v) $\frac{d}{dt}[FGH] = \left(\frac{dF}{dt} GH\right) + \left(F \frac{dg}{dt} H\right) + \left(FG \frac{dH}{dt}\right)$

vi) $\frac{d}{dt}[(F \times G) \times H] = \frac{dF}{dt} \times G \times H + F \times \frac{dG}{dt} \times H + (F \times G) \times \frac{dH}{dt}$

ii.

Remark: If F(t) has a constant magnitude, then F. dF = 0
dt

If F(t) has constant (fixed) direction, then F x dF = 0
Dt

Example: If A= $5t^2$I + tJ – t^3K, B=sintI - cos(t)J then
Find i) d/dt(A.B)
ii) d/dt(AxB)

Solution: i) d/dt(A.B) = A.(dB/dt) + (dA/dt).B
= ($5t^2$I+tJ-t^3K)[costI –(-sint)J] + (10tI+J-$3t^2$K)(sintI - costJ)
= ($5t^2$cost-tsint) + (10tsint-cost)
= $5t^2$ cos(t) + 11tsin(t) – cos(t).

ii) d/dt(AXB) = Ax(dB/dt) + (dA/dt)xB
= ($5t^2$I+tJ-t^3K)x(costI +sintJ) + (10tI+J-$3t^2$K)x(sintI -costJ)
= (t^3sint-$3t^2$cost)I – t^2(tcost+3sint)J + [($5t^2$-1)sint-11tcost]K

Derivative of Polar Functions

Let r = r(?) represent a polar curve, then
dy = dy/d? = r'sin? + rcos?

dx dx/d? r'cos? – rsin?

Since x=rcos?
dx/d? = r(-sin?) + cos?. r' = r'cos? – rsin?
y = rsin?
dy/d? = rcos? + sin?. r' = r'sin? + rcos?

dy = dy/d? = r'sin? + rcos?

dx dx/d? r'cos? – rsin?

Example: Find the derivative of r = ?cos?

Example: dy/dx = [?(-sin?) + cos?] sin? + ? cos? (cos?)

[?(-sin?) + cos?] cos? – ? cos? sin?

= -?sin2? + cos? sin? + ?cos2?

-?sin? cos? + cos2? – ?cos? sin?

= cos? sin? + ?(cos2? – sin2?)

cos2? - 2? cos? sin?

= cos? sin?+ ?cos2?

cos2? - 2? cos? sin?

Example: Find the slope of the tangent line to the unit circle x= cost, y = sint (0?t?2?) at the point t = ?/6

Solution: The slope at a general point on the circle is dy/dx

dy/dx = dy/dt = cost = -cot t

dx/dt -sint

Thus, the slope at t = ?/6 is

$dy/dx]_{t=?/6}$ = -cot ?/6 = -?3

Example: Find the slope of the tangent line to the circle r = 4cos? at the point where ? = ?/4

Solution:

$$dy/dx = \frac{4\cos 2\theta - 4\sin 2\theta}{-8\sin\theta\cos\theta} = \frac{4\cos 2\theta}{-4\sin 2\theta} = -\cot 2\theta$$

Thus, at the point where ? = ?/4 the slope of the tangent line is dy/dx]?=?/4 = -cot?/2 = 0 which implies that the circle has a horizontal tangent line at the point where ? = ?/4

Now try it yourself! Should you still need any help, click here to schedule live online session with e Tutor!

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Reference links:

- http://en.wikipedia.org/wiki/Derivative
- http://en.wikipedia.org/wiki/Polar_curve
- http://en.wikipedia.org/wiki/Slope

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