## DERIVATIVES OF POLAR AND VECTOR FUNCTION

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## Differentiation of Vectors



If a vector $R$ varies continuously as a scalar variable $t$ changes, then $R$ is said to be a function of $t$ and is written as $R=F(t)$. Just as in scalar calculus, we define derivative of a vector function $R=F(t)$ as
$\operatorname{Lim} F(t+\delta t)-F(t)$
$\overline{\delta t} \begin{array}{lll} & 0 & \delta t\end{array}$ and write it as $\mathrm{dR} / \mathrm{dt}$ or $\mathrm{dF} / \mathrm{dt}$ or $\mathrm{F}^{\prime}(\mathrm{t})$.

General rules of differentiation are similar to those of ordinary calculus provided the order of factors in vector products is maintained. Thus, if ?, F, G, H are scalar and vector functions of a scalar variable $t$, we have

$$
\text { i) } \frac{\mathrm{d}}{\mathrm{dt}}[\mathrm{~F}+\mathrm{G}-\mathrm{H}]=\frac{\mathrm{df}}{\mathrm{dt}}+\frac{\mathrm{dG}}{\mathrm{dt}}-\frac{\mathrm{dH}}{\mathrm{dt}}
$$

ii) $d[F \Phi]=F d \Phi+\Phi d F$
dt $\quad \overline{d t} \quad \overline{d t}$
$\frac{\text { iii) }}{\frac{d}{d t}}[F . G]=\frac{F . d G}{d t}+\frac{d F}{d t}$
i.
iv) $d(F \times G)=\underset{d t}{F d G}+d F \times G$


ii.

If $F(t)$ has constant (fixed) direction, then $F \times d F=0$
Dt

Example: If $A=5 t^{2} I+t J-t^{3} K, B=\sin t I-\cos (t) J$ then
Find i) d/dt(A.B)
ii) $d / d t(A x B)$

Solution: i) $\mathrm{d} / \mathrm{dt}(\mathrm{A} \cdot \mathrm{B})=\mathrm{A} \cdot(\mathrm{dB} / \mathrm{dt})+(\mathrm{dA} / \mathrm{dt}) \cdot \mathrm{B}$

$$
\begin{aligned}
& =\left(5 \mathrm{t}^{2} \mathrm{I}+\mathrm{tJ}-\mathrm{t}^{3} \mathrm{~K}\right)[\operatorname{costI}-(-\sin \mathrm{t}) \mathrm{J}]+\left(10 \mathrm{tI}+\mathrm{J}-3 \mathrm{t}^{2} \mathrm{~K}\right)(\sin \mathrm{I}-\cos \mathrm{J}) \\
& =\left(5 \mathrm{t}^{2} \cos -\mathrm{t} \sin \mathrm{t}\right)+(10 \mathrm{tsint}-\cos \mathrm{t}) \\
& =5 \mathrm{t}^{2} \cos (\mathrm{t})+11 \mathrm{tsin}(\mathrm{t})-\cos (\mathrm{t}) .
\end{aligned}
$$

ii) $\mathrm{d} / \mathrm{dt}(\mathrm{AXB})=\mathrm{Ax}(\mathrm{dB} / \mathrm{dt})+(\mathrm{dA} / \mathrm{dt}) \mathrm{xB}$

$$
\begin{aligned}
& =\left(5 \mathrm{t}^{2} \mathrm{I}+\mathrm{tJ}-\mathrm{t}^{3} \mathrm{~K}\right) \mathrm{x}(\cos \mathrm{II}+\sin \mathrm{J})+\left(10 \mathrm{tI}+\mathrm{J}-3 \mathrm{t}^{2} \mathrm{~K}\right) \mathrm{x}(\sin \mathrm{I}-\cos \mathrm{J}) \\
& =\left(\mathrm{t}^{3} \sin t-3 \mathrm{t}^{2} \cos \mathrm{t}\right) \mathrm{I}-\mathrm{t}^{2}(\mathrm{t} \cos \mathrm{t}+3 \sin \mathrm{t}) \mathrm{J}+\left[\left(5 \mathrm{t}^{2}-1\right) \sin \mathrm{t}-11 \mathrm{tcos} \mathrm{t}\right] \mathrm{K}
\end{aligned}
$$

## Derivative of Polar Functions

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Let \(r=r(?)\) represent a polar curve, then
    \(\mathrm{dy}=\mathrm{dy} / \mathrm{d} ?=\mathrm{r}\) 'sin\(?+\mathrm{rcos} ?\)
        dx dx/d? r'cos? - rsin?
```

Since $x=r \cos$ ?
$\mathrm{dx} / \mathrm{d} ?=\mathrm{r}(-\sin ?)+\cos ? . \mathrm{r}^{\prime}=\mathrm{r}^{\prime} \cos ?-\mathrm{r} \sin ?$
$y=r \sin$ ?
$\mathrm{dy} / \mathrm{d} ?=\mathrm{rcos} ?+\sin ? . \mathrm{r}^{\prime}=\mathrm{r}$ 'sin$?+\mathrm{rcos} ?$
$\mathrm{dy}=\mathrm{dy} / \mathrm{d} ?=\mathrm{r}$ 'sin $?+\mathrm{rcos}$ ?
dx dx/d? r'cos? - rsin?
Example: Find the derivative of $r=? \cos$ ?
Example: $\mathrm{dy} / \mathrm{dx}=[?(-\sin ?)+\cos ?] \sin ?+? \cos ?(\cos ?)$

$$
\begin{aligned}
& {[?(-\sin ?)+\cos ?] \cos ?-? \cos ? \sin ? } \\
= & -? \sin 2 ?+\cos ? \sin ?+? \cos 2 ?
\end{aligned}
$$

$$
-? \sin ? \cos ?+\cos 2 ?-? \cos ? \sin ?
$$

$$
=\cos ? \sin ?+?(\cos 2 ?-\sin 2 ?)
$$

$$
\begin{aligned}
& \cos 2 ?-2 ? \cos ? \sin ? \\
= & \cos ? \sin ?+? \cos 2 ?
\end{aligned}
$$

$$
\cos 2 ?-2 ? \cos ? \sin ?
$$

Example: Find the slope of the tangent line to the unit circle $\mathrm{x}=\operatorname{cost}, \mathrm{y}=\operatorname{sint}(0 ? \mathrm{t} ? 2$ ?) at the point $\mathrm{t}=? / 6$
Solution: The slope at a general point on the circle is $d y / d x$
$\mathrm{dy} / \mathrm{dx}=\mathrm{dy} / \mathrm{dt}=\cos \mathrm{t}=-\cot \mathrm{t}$

$$
\mathrm{dx} / \mathrm{dt}-\sin t
$$

Thus, the slope at $t=? / 6$ is
$\mathrm{dy} / \mathrm{dx}] \mathrm{t}=? / 6=-\cot ? / 6=-? 3$
Example: Find the slope of the tangent line to the circle $r=4 \cos ?$ at the point where $?=? / 4$

$$
\mathrm{dy} / \mathrm{dx}=\stackrel{4 \cos 2 \theta}{1}-4 \sin 2 \theta=4 \cos 2 \theta=-\cot 2 \theta
$$

Solution: $\quad-8 \sin \theta \cos \theta \quad-4 \sin 2 \theta$
Thus, at the point where $?=? / 4$ the slope of the tangent line is $d y / d x] ?=? / 4=-\cot ? / 2=0$ which implies that the circle has a horizontal tangent line at the point where ? = ?/4

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## Reference links:

- http://en.wikipedia.org/wiki/Derivative
- http://en.wikipedia.org/wiki/Polar_curve
- http://en.wikipedia.org/wiki/Slope

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