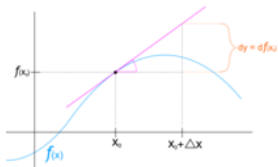


Differentials, Errors and Approximations

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Approximations



Here we will use [differentials](#) to approximate values of certain quantities.

Let $f: D \rightarrow \mathbb{R}$, $D \subset \mathbb{R}$, be a given function and $y = f(x)$. Let Δx be a small increment in x , so that Δy will be the corresponding increment in y then,

Δy is given by the formula, $\Delta y = f(x + \Delta x) - f(x)$.

Differentials

- i) The differential of x is denoted by dx and it is defined by $dx = \Delta x$
- ii) The differential of y , denoted by dy , is defined by $dy = f'(x) dx$ or $dy = (dy/dx) \Delta x$

Approximate Value of irrationals

For finding the [approximate value](#) of irrationals, first we have to take the integral part or bigger number as ' x ' and the decimal part or smaller number as Δx . Here, we take $\Delta y = \Delta y$ and for evaluating Δy use the formula $\Delta y = (dy/dx) \Delta x$.

For example: Use differentials to approximate $(25)^{1/3}$

$$(25)^{1/3} = (27 + (-2))^{1/3}$$

Take $x = 27$, which is a perfect cube and $\Delta x = -2$

Let $y = x^{1/3}$

$$y + \Delta y = (x + \Delta x)^{1/3}$$

$$\Delta y = (x + \Delta x)^{1/3} - x^{1/3}$$

$$= (27 + (-2))^{1/3} - (27)^{1/3}$$

$$\Delta y = (25)^{1/3} - 3 \dots\dots\dots(i)$$

$$\Delta y = dy = (dy/dx) \Delta x$$

$$\begin{aligned}
 &= \frac{1(-2)}{3x^{2/3}} \quad \left[\frac{d(x^{1/3})}{dx} = \frac{1}{3x^{2/3}} \right] \\
 &= \frac{-2}{3(27)^{2/3}} \\
 &= \frac{-2}{27}
 \end{aligned}$$

$$= -0.074$$

Equation (i) becomes $-0.074 = (25)^{1/3} - 3$

$$-0.074 + 3 = (25)^{1/3}$$

Hence $(25)^{1/3} = 2.926$

Approximate value of a function

In this case a function $f(x)$ will be given and we have to find the value of the function at a given decimal number. Here also, we take the integral part as 'x' and decimal part as Δx . The formula is, $f(x+\Delta x) = \Delta y + f(x)$, where $\Delta y = f'(x)\Delta x$

For example: Find the approximate value of $f(3.02)$ where $f(x) = 3x^2 + 5x + 3$

Let $x = 3$ and $\Delta x = 0.02$,

$$f(x) = 3x^2 + 5x + 3$$

$$f'(x) = 6x + 5$$

$$\Delta y = f'(x)\Delta x$$

$$= (6x + 5)(0.02)$$

$$= (6 \times 3 + 5)(0.02)$$

$$= 23 \times 0.02$$

$$= 0.46$$

$$f(3 + .02) = 0.46 + f(3)$$

$$= 0.46 + [3(3)^2 + 5(3) + 3]$$

$$= 0.46 + 45$$

$$f(3.02) = 45.46$$

Approximate error

Here we learn to find the approximate error in volume, surface area etc caused by the error in taking radius.

For example: If the radius of a sphere is measured as 9m with an error of 0.03m, then find the approximate in calculating its surface area.

Solution: $r=9\text{m}$ and $\Delta r=0.03\text{m}$

$$V = \frac{4\pi r^3}{3}$$

$$\frac{dV}{dr} = 4\pi r^2$$

$$\begin{aligned}dV &= \left(\frac{dV}{dr} \Delta r \right) \\ &= 4\pi r^2 (\Delta r) \\ &= 4\pi \times 81 \times 0.03 \\ &= 9.72\pi \text{m}^3\end{aligned}$$

Thus the approximate error in calculating the volume is $9.72\pi \text{m}^3$

Now try it yourself! Should you still need any help, [click here](#) to schedule live online session with e Tutor!

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Reference Links:

- http://en.wikipedia.org/wiki/Differential_of_a_function
- http://www.mathwords.com/a/approximation_by_differentials.htm
- <http://en.wikibooks.org/wiki/Algebra/Functions>
- http://en.wikipedia.org/wiki/Approximation_error

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