Differentials, Errors and Approximations

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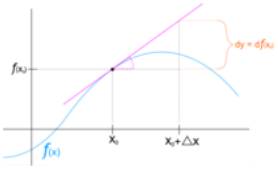
Approximations



Here we will use differentials to approximate values of certain quantities.
Let f: D ---- R, DCR, be a given function and y = f(x). Let ? x be a small increment in x, so that ? y will be the corresponding increment in y then,
? y is given by the formula, ? y = f(x + ? x) - f(x).

Differentials

i) The differential of x is denoted by dx and it is defined by dx = ? x
ii) The differential of y, denoted by dy, is defined by dy = f'(x) dx or dy = (dy/dx) ?x

Approximate Value of irrationals

For finding the approximate value of irrationals, first we have the take the integral part or bigger number as 'x' and the decimal part or smaller number as ?x. Here, we take dy = ?y and for evaluating dy use the formula dy = (dy/dx) ?x.
For example: Use differentials to approximate $(25)^{1/3}$
$(25)^{1/3} = (27 + (-2))^{1/3}$
Take x = 27, which is a perfect cube and ?x = -2
Let $y=x^{1/3}$
$y + ?y = (x + ?x)^{1/3}$
$?y = (x + ?x)^{1/3} - x^{1/3}$
$= (27 + (-2))^{1/3} - (27)^{1/3}$
$?y = (25)^{1/3} - 3$ ……..(i)
?y = dy = (dy/dx) ?x

$$= \frac{1(-2)}{3x^{2/3}} \qquad \left[\frac{d}{dx}(x^{1/3}) = \frac{1}{3x^{2/3}}\right]$$

$$= \frac{-2}{3(27)^{2/3}}$$

$$= \frac{-2}{27}$$

=-0.074

Equation (i) becomes $-0.074 = (25)^{1/3} - 3$

$-0.074 + 3 = (25)^{1/3}$

Hence $(25)^{1/3} = 2.926$

Approximate value of a function

In this case a function f(x) will be given and we have to find the value of the function at a given decimal number. Here also, we take the integral part as 'x' and decimal part as ?x. The formula is, f(x+?x)=?y + f(x), where ?y=f'(x)?x

For example: Find the approximate value of f(3.02) where $f(x)=3x^2+5x+3$

Let x = 3 and ?x = 0.02,

$f(x) = 3x^2 + 5x + 3$

f'(x) = 6x + 5

?y = f'(x) ?x

= (6x + 5) (0.02)

= (6 x 3 + 5) (0.03)

= 2 3x 0.03

= 0.46

f (3 + .02) = 0.46 + f(3)

=0.46 + [3(3)2+5(3)+3]

=0.46 + 45

f(3.02) = 45.46

Approximate error

Here we learn to find the approximate error in volume, surface area etc caused by the error in taking radius.

For example: If the radius of a sphere is measured as 9m with an error of 0.03m, then find the approximate in calculating its surface area.

Solution: r=9m and ?r=0.03m

$$V = \frac{4\prod r^3}{3}$$

$$\frac{dV}{dr} = 4\prod r^2$$

$$dV = \left(\frac{dV}{dr}\right)\Delta r$$

$$= 4\prod r^2 (\Delta r)$$

$$= 4\prod x81x0.03$$

$$= 9.72\prod m^3$$

Thus the approximate error in calculating the volume is 9.72?m^3

Now try it yourself! Should you still need any help,click here to schedule live online session with e Tutor!

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Reference Links:

- http://en.wikipedia.org/wiki/Differential_of_a_function
- http://www.mathwords.com/a/approximation_by_differentials.htm
- http://en.wikibooks.org/wiki/Algebra/Functions
- http://en.wikipedia.org/wiki/Approximation_error

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