DIFFERENTIATION - I

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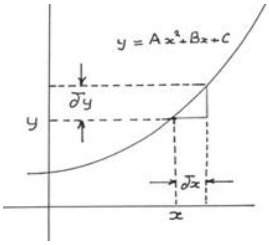
Introduction



In this topic, we are dealing with different types of differentiation. Different methods are derivatives of implicit functions, exponential functions, and logarithmic functions.

Derivatives of Implicit Functions

When a relationship between x and y is expressed in a way that it is easy to solve for y and write y=f(x), we say that y is given as an explicit function of 'x', otherwise it is implicit that y is a function of x and we say that the relationship gives function implicitly.
For example: 1) 2x+3y+5=0
2) x+cosxy+3=0
In the above examples, first one is an explicit function, since it can be easily written as y= (-2x-5)/3 and is easy to solve for y. But in the second one, we find difficult to separate y from that function, so it is an implicit function. In the case of an implicit function, we directly differentiate the given function with respect to x and find dy/dx.
Find dy/dx for the function $ax + by^2 = \cos y$
Differentiating with respect to x we get,
a. 1+b. 2ydy/dx= -siny dy/dx
(2by+siny)dy/dx = -a

$$dy/dx = \frac{-a}{2by+siny}$$

Exponential Function

The exponential function with positive base b>1 is the function $y=b^x$
The domain of the exponential function is R, the set of real numbers and the range is set of all positive real numbers.
Exponential function with base 10 is called the common exponential function, which is given by $y=10^x$
Using 'e' as the base, we get an extremely important exponential function $y=e^x$ which is called natural exponential function
The derivative of e^x is e^x itself

$$\frac{d(e^x)=e^x}{dx}$$

The derivative of b^x is given by,

$$\frac{d}{dx}(b^x) = b^x \log b$$

Logarithmic Function

Let b>1 and if $b^x=a$, then we say logarithm of 'a' to the base 'b' is 'x' and it is denoted by $\log_b a=x$. This function is defined from R^+ to R.
The domain of logarithmic function is R^+ and the range is R, set of all real numbers.
If the base b=10, then we say it as common logarithm and if b=e then it is natural logarithm.

The derivative of log function is given by

$$\frac{d}{dx}(\log x) = \frac{1}{x}$$

Logarithmic Differentiation

Logarithmic differentiation is applicable for the functions in the form
$y= [u(x)]^{v(x)}$, taking log on both sides we get,

$$\log y = \log[u(x)]^{v(x)}$$
$$= v(x) \log[u(x)]$$

Using Chain Rule, we may differentiate this to get,

$$\frac{1}{y}\frac{dy}{dx} = v(x)\frac{1}{u(x)}u'(x) + v'(x)\log[u(x)]$$
$$\frac{dy}{dx} = y\left(\frac{v(x)}{u(x)}u'(x) + v'(x)\log[u(x)]\right)$$

The main point to be noted in this method is that 'y' and u(x) must always be positive otherwise their logarithms are not defined.
This process of differentiation is known as Logarithmic Differentiation.

Laws of logarithms

1) $\log_b pq = \log_b p + \log_b q$
2) $\log_b(p/q) = \log_b p - \log_b q$
3) $\log_b p^q = q\log_b p$ [Power Rule]
4) $\log_a p = \frac{\log_b p}{\log_b a}$ [Change of Base]

Example: Differentiate $x^{\sin x}$, x>0 w.r.t x
Solution: Let $y = x^{\sin x}$
Taking log on both sides,
$\log y = \log x^{\sin x}$
$\log y = \sin x \log x$
Differentiating,

$$\frac{1}{y}\frac{dy}{dx} = \sin x \frac{1}{x} + \log x \cos x$$
$$\frac{dy}{dx} = y\left(\frac{\sin x}{x} + \cos x \log x\right)$$
$$= x^{\sin x}\left(\sin x/x + \cos x \log x\right)$$

Now try it yourself! Should you still need any help,click here to schedule live online session with e Tutor!

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Reference Links :

- http://en.wikipedia.org/wiki/Implicit_and_explicit_functions
- http://en.wikipedia.org/wiki/Logarithmic_differentiation
- http://en.wikipedia.org/wiki/Exponential_function
- http://www.intmath.com/functions-and-graphs/2a-domain-and-range.php
- http://en.wikipedia.org/wiki/Logarithm
- http://en.wikipedia.org/wiki/Derivative

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