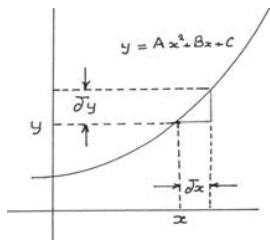


DIFFERENTIATION - I

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Introduction



In this topic, we are dealing with different types of [differentiation](#). Different methods are derivatives of implicit functions, exponential functions, and logarithmic functions.

Derivatives of Implicit Functions

When a relationship between x and y is expressed in a way that it is easy to solve for y and write $y=f(x)$, we say that y is given as an [explicit function](#) of ' x ', otherwise it is implicit that y is a function of x and we say that the relationship gives function implicitly.

For example: 1) $2x+3y+5=0$

2) $x+\cos xy+3=0$

In the above examples, first one is an explicit function, since it can be easily written as $y= (-2x-5)/3$ and is easy to solve for y . But in the second one, we find difficult to separate y from that function, so it is an implicit function. In the case of an implicit function, we directly differentiate the given function with respect to x and find dy/dx .

Find dy/dx for the function $ax + by^2 = \cos y$

Differentiating with respect to x we get,

$$a + b \cdot 2y \frac{dy}{dx} = -\sin y \frac{dy}{dx}$$

$$(2by + \sin y) \frac{dy}{dx} = -a$$

$$\frac{dy}{dx} = \frac{-a}{2by + \sin y}$$

Exponential Function

The [exponential function](#) with positive base $b>1$ is the function $y=b^x$

The [domain](#) of the exponential function is \mathbb{R} , the set of real numbers and the range is set of all positive real numbers.

Exponential function with base 10 is called the common exponential function, which is given by $y=10^x$

Using 'e' as the base, we get an extremely important exponential function $y=e^x$ which is called natural exponential function

The derivative of e^x is e^x itself

$$\frac{d(e^x)}{dx} = e^x$$

The derivative of b^x is given by,

$$\frac{d(b^x)}{dx} = b^x \log b$$

Logarithmic Function

Let $b > 1$ and if $b^x = a$, then we say logarithm of 'a' to the base 'b' is 'x' and it is denoted by $\log_b a = x$. This function is defined from \mathbb{R}^+ to \mathbb{R} .

The domain of [logarithmic function](#) is \mathbb{R}^+ and the range is \mathbb{R} , set of all real numbers.

If the base $b=10$, then we say it as common logarithm and if $b=e$ then it is natural logarithm.

The derivative of log function is given by

$$\frac{d(\log x)}{dx} = \frac{1}{x}$$

Logarithmic Differentiation

[Logarithmic differentiation](#) is applicable for the functions in the form

$y = [u(x)]^{v(x)}$, taking log on both sides we get,

$$\begin{aligned} \log y &= \log [u(x)]^{v(x)} \\ &= v(x) \log [u(x)] \end{aligned}$$

Using Chain Rule, we may differentiate this to get,

$$\begin{aligned} \frac{1}{y} \frac{dy}{dx} &= v(x) \frac{1}{u(x)} u'(x) + v'(x) \log [u(x)] \\ \frac{dy}{dx} &= y \left(\frac{v(x) u'(x) + v'(x) \log [u(x)]}{u(x)} \right) \end{aligned}$$

The main point to be noted in this method is that 'y' and $u(x)$ must always be positive otherwise their logarithms are not defined.

This process of differentiation is known as Logarithmic Differentiation.

Laws of logarithms

- 1) $\log_b pq = \log_b p + \log_b q$
- 2) $\log_b (p/q) = \log_b p - \log_b q$
- 3) $\log_b p^q = q \log_b p$ [Power Rule]
- 4) $\log_a p = \log_b p \cdot \log_b a$ [Change of Base]

$$\log_b a$$

Example: Differentiate $x^{\sin x}$, $x > 0$ w.r.t x

Solution: Let $y = x^{\sin x}$

Taking log on both sides,

$$\log y = \log x^{\sin x}$$

$$\log y = \sin x \log x$$

Differentiating,

$$\frac{1}{y} \frac{dy}{dx} = \sin x \frac{1}{x} + \log x \cos x$$

$$\frac{dy}{dx} = y \left(\frac{\sin x}{x} + \cos x \log x \right)$$

$$= x^{\sin x} \left(\frac{\sin x}{x} + \cos x \log x \right)$$

Now try it yourself! Should you still need any help,[click here](#) to schedule live online session with e Tutor!

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Reference Links :

- http://en.wikipedia.org/wiki/Implicit_and_explicit_functions
- http://en.wikipedia.org/wiki/Logarithmic_differentiation
- http://en.wikipedia.org/wiki/Exponential_function
- <http://www.intmath.com/functions-and-graphs/2a-domain-and-range.php>
- <http://en.wikipedia.org/wiki/Logarithm>
- <http://en.wikipedia.org/wiki/Derivative>

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