DIFFERENTIATION

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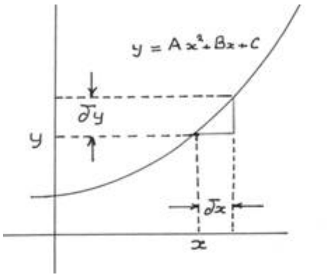
Introduction



Let ‘f’ be a given function, then the derivative of ‘f’ is denoted by f’(x) and is defined as, $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

The process of finding derivative is called differentiation.
The derivative of a function can be denoted is different ways, they are
y’, y1, dy / dx etc.
The derivative of a function at ‘c’ is denoted as f’(c) and is defined as

$$f'(c) = \lim_{h \to 0} \frac{f(c+h) - f(c)}{h}$$

The process of finding the derivative using definition is called the first principle of differentiation

Example: Using 1st principle of differentiation, find the derivative of $(x+2)^2$
Let $f(x) = (x + 2)^2$, $f(x + h) = (x + h + 2)^2$

$$= \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x + h + 2)^2 - (x + 2)^2}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + h^2 + 4 + 2xh + 4h + 4x - x^2 - 4x - 4}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{h^2 + 2xh + 4h}{h}$$

$$= \lim_{h \to 0} h + 2x + 4$$

$= 0 + 2x + 4$
$= 2(x + 2)$

List of derivatives of certain standard functions

S. No.	f (x)	f' (x)
1	x^n	nx^{n-1}
2	Sin x	Cos x
3	Cos x	-Sin x
4	Tan x	Sec 2x
5	Cot x	-Cosec2x
6	Sec x	Sec x Tan x
7	Cosec x	-Cosec x Cot x
8	log x	1/x
9	a constant	Zero
10	e^x	e^x
11	a^x	$a^x \log a$
12	$\sqrt{x}$	$1 / (2\sqrt{x})$

Product Rule of Differentiation

If 'u' and 'v' are functions of 'x' then

$$\frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

Derivative of product of two functions is "(first function) x (derivative of second) + (second function) x (derivative of first)"
If u, v and w are functions of 'x' then

$$\frac{d}{dx}(uvw) = uv\frac{dw}{dx} + uw\frac{dv}{dx} + vw\frac{du}{dx}$$

Quotient Rule of Differentiation

If ‘u’ and ‘v’ are functions of ‘x’ and v?0, then quotient rule is

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\left(\frac{du}{dx}\right) - u\left(\frac{dv}{dx}\right)}{v^2}$$

$$= \frac{v\,u' - u\,v'}{v^2}$$

Important Notes:
(i) (u ± v)’ = u’ ± v’
(ii) If a function ‘f’ is differentiable at a point ‘c’ then it is continuous at that point.
(iii) Every differentiable function is continuous.

Chain Rule of Differentiation

Chain Rule is applicable only for the composition of functions. Let ‘y’ be a composition of two functions ‘f’ and ‘g’.

y = f o g = f [g(x)]

Take y= f(u) where u = g(x) so that we can find

dy/du and du/dx [Since ‘y’ is a function of ‘u’ we get dy/du and ‘u’ is a function of ‘x’ we get du/dx]

Hence $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

If ‘y’ is the composition of three functions ‘f’, ‘g’ and ‘u’ then

y= f o g o u
= f {g[u(x)]}

Take v=u(x), t=g(v) and y=f(t)

Find dv/dx, dt/dv and dy/dt

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dv} \times \frac{dv}{dx}$$

Example: Find the derivative of Cos (3x + 5)
Solution: Let y = Cos (3x + 5)
Take y = Cos (u) where u = 3x + 5

$$\frac{dy}{du} = -\text{Sin}(u) \text{ and } \frac{du}{dx} = 3$$

$$\frac{dy}{dx} = \left(\frac{dy}{du}\right) \times \left(\frac{du}{dx}\right) = -3\ \text{Sin}(u)$$

$$\frac{dy}{dx} = -3\ \text{Sin}(3x+5) \qquad [u = 3x + 5]$$

Now try it yourself! Should you still need any help, click here to schedule live online session with e Tutor!

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Reference Links :

- http://en.wikipedia.org/wiki/Derivative
  http://en.wikipedia.org/wiki/Quotient_rule
- http://en.wikipedia.org/wiki/Chain_rule
  http://en.wikipedia.org/wiki/Function_%28mathematics%29
- http://en.wikipedia.org/wiki/Function_composition

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