# DIFFERENTIATION - II

Created: Thursday, 10 November 2011 10:04 | Published: Thursday, 10 November 2011 10:04 | Written by Super User | Print

# Introduction

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differentiation-ii Here we will discuss about the derivatives of Parametric Functions, Inverse trigonometric functions and second order derivatives. For finding the derivatives of parametric and inverse trigonometric functions we apply Chain Rule of Differentiation. In the case of inverse trigonometric functions we have to substitute correct values to the variable in the given function so as to reduce it into simplest form. For this substitution, we must be aware of the trigonometric identities.

## Derivative of Parametric Functions

In some cases the relation between two variables is neither explicit nor implicit, but some link of a third variable with each of the two variables. The third variable is called the parameter. In other words, a relation expressed between two variables x and y in the form x=f(t), y=g(t) is said to be parametric form with ‘t’ as a parameter.

To find the derivative of parametric functions we proceed in this way,
Let x=f(t) and y=g(t), then find dx/dt and dy/dt,

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

Example: If x=acos? and y= bcos?, find dy/dx
Solution: Given x=acos? and y=bcos?

$$\frac{dx}{d\theta} = a(-\sin\theta) \quad \frac{dy}{d\theta} = b(-\sin\theta)$$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{-b\sin\theta}{-a\sin\theta}$$

$$\frac{dy}{dx} = \frac{b}{a}$$

It must be noted that in finding the derivatives of parametric functions, dy/dx is expressed in terms of parameter only without directly involving the main variable ‘x’ and ‘y’.

## Derivatives of inverse trigonometric functions

We have already learnt about inverse trigonometric functions. They are continuous functions. Now, we will find the derivative of inverse trigonometric functions using Chain Rule.

1. Let $f(x)=\sin^{-1}x$, we are finding its derivative
Take $y= \sin^{-1}x$, x=siny
Differentiating both sides w.r.t x, we get

$1 = \cos y \,(dy/dx)$

$$\frac{dy}{dx} = \frac{1}{\cos y}$$

$$= \frac{1}{\sqrt{1-\sin^2 y}}$$

$$= \frac{1}{\sqrt{1-x^2}} \qquad [\, x=\sin y]$$

Hence $\mathbf{\dfrac{d(\sin^{-1}x)}{dx} = \dfrac{1}{\sqrt{1-x^2}}}$

2. Let $y=\cos^{-1}x$, $x=\cos y$

Differentiating both sides w.r.t x we get,

$1 = -\sin y \,(dy/dx)$

$$\frac{dy}{dx} = \frac{-1}{\sin y}$$

$$= \frac{-1}{\sqrt{1-\cos^2 y}}$$

$$= \frac{-1}{\sqrt{1-x^2}} \qquad [x=\cos y]$$

Hence $\mathbf{\dfrac{d(\cos^{-1}x)}{dx} = \dfrac{1}{\sqrt{1-x^2}}}$

Similarly we can find the derivatives of remaining four inverse trigonometric functions.

# List of the derivatives of inverse trigonometric functions

The following table gives the derivatives of the inverse trigonometric functions

| Sl. No: | f(x) | f'(x) |
|---|---|---|
| 1 | $\sin^{-1}x$ | $\frac{1}{\sqrt{1-x^2}}$ |
| 2 | $\cos^{-1}x$ | $\frac{-1}{\sqrt{1-x^2}}$ |
| 3 | $\tan^{-1}x$ | $\frac{1}{1+x^2}$ |
| 4 | $\cot^{-1}x$ | $\frac{-1}{1+x^2}$ |
| 5 | $\sec^{-1}x$ | $\frac{1}{x\sqrt{x^2-1}}$ |
| 6 | $\operatorname{cosec}^{-1}x$ | $\frac{-1}{x\sqrt{x^2-1}}$ |

# Second order derivatives

If we differentiate a function two times, we get second order derivative. By higher order derivatives, we mean second, third, fourth ……… nth order derivatives. When a function is differentiated two times we get second derivative, three times third derivative and so on.  But in this topic, we deal with second order derivatives.

Second order derivatives are denoted by $d^2y/dx^2$, $f''(x)$, $y''$, $y_2$ or $D^2y$.

Example 1: Find the second derivative of $x^{20}$

Solution: Let $f(x)=x^{20}$

$f'(x)=20x^{19}$

$f''(x)= 20 * 19x^{18}$

$= 380x^{18}$

Example 2: If $y = (\tan^{-1}x)^2$, show that $(x^2+1)^2y_2+2x(x^2+1)y_1=2$

Solution: $y= (\tan^{-1}x)^2$

$$y_1= 2(\tan^{-1}x)\frac{1}{1+x^2} \quad \text{[Differentiating 1}^{st}\text{ time]}$$

$$(1+x^2)\, y_1 = 2\tan^{-1}x \quad \text{[cross multiplying } 1+x^2\text{]}$$

$$(1+x^2)\, y_2 + y_1\,(2x) = 2\left(\frac{1}{1+x^2}\right) \quad \text{[Differentiating 2}^{nd}\text{ time]}$$

$(1+x^2)^2\, y_2 + 2x\,(1+x^2)\, y_1 = 2$  [cross multiplying $1+x^2$]

$(x^2 +1)^2\, y_2 + 2x(x^2+1)y_1 = 2$

Now try it yourself!  Should you still need any help, click here to schedule live online session with e Tutor!

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## Reference Links:

- http://en.wikipedia.org/wiki/Parametric_equation
  http://en.wikipedia.org/wiki/Inverse_trigonometric_functions
- http://en.wikipedia.org/wiki/List_of_trigonometric_identities
  http://en.wikibooks.org/wiki/Calculus/Higher_Order_Derivatives
  http://en.wikipedia.org/wiki/Parameter#Parameters_in_mathematics_and_science

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