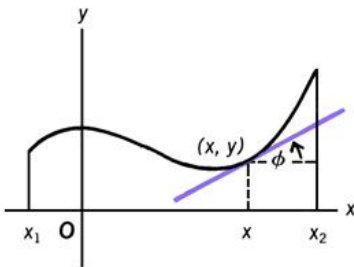


# GRAPHICAL AND ANALYTICAL REPRESENTATION OF DERIVATIVE

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## Introduction



The main idea of [derivative](#) is that of rate of change of a function. The primary mathematical tool that is used to calculate rate of change and the slopes of curves is derivative. Slope of the graph of  $y=f(x)$  at  $x=x_0$  is given by

$$m_{\text{curve}} = \lim_{x_1 \rightarrow x_0} \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

The ratio  $\frac{f(x_1) - f(x_0)}{x_1 - x_0}$  is called a [difference quotient](#).

The difference quotient can also be interpreted as the average rate of change of  $f(x)$  over the interval  $[x_0, x_1]$  and its limit as  $x_1 \rightarrow x_0$  is the instantaneous rate of change of  $f(x)$  at  $x=x_0$ .

## Derivative of a function

Suppose that  $x_0$  is a number in the domain of a function  $f$  then the derivative of 'f' at  $x=x_0$  and is denoted by  $f'(x_0)$  and is defined as

$$f'(x_0) = \lim_{x_1 \rightarrow x_0} \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

If the limit of the difference quotient exists,  $f'(x_0)$  is the [slope](#) of the graph of 'f' at the point  $x=x_0$ . If this limit does not exist, then the slope of the graph of 'f' is undefined at  $x=x_0$

Example: Find the slope of the function  $y=x^2+1$  at the point (2, 5)

Solution: Slope of the curve at the point (2, 5) is given by

$$f'(2) = \lim_{x_1 \rightarrow 2} \frac{f(x_1) - f(2)}{x_1 - 2} = \lim_{x_1 \rightarrow 2} \frac{(x_1^2 + 1) - 5}{x_1 - 2} = \lim_{x_1 \rightarrow 2} \frac{x_1^2 - 4}{x_1 - 2}$$

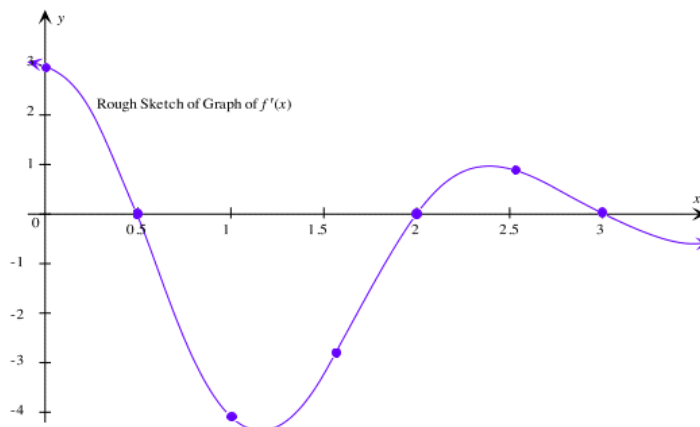
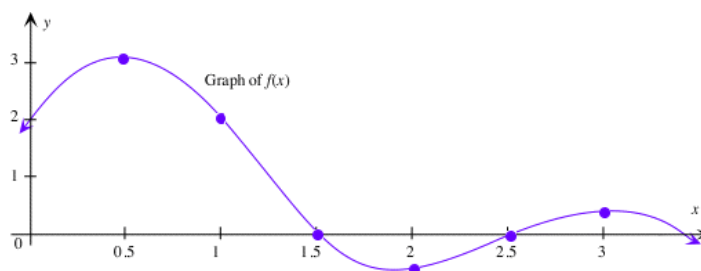
$$= 2 + 2 = 4$$

## Graphical representation of Derivative

Suppose that  $x_0$  is a number in the domain of a function 'f'. If

$$f'(x_0) = \lim_{x_1 \rightarrow x_0} \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

then we define the tangent line to the graph of  $f$  at the point  $P(x_0, f(x_0))$  to be the line whose equation is  $y - f(x_0) = f'(x_0)(x - x_0)$ . Below is the graph of a function  $f(x)$  and a rough sketch of  $f'(x)$ . The graph of  $f'(x)$  represents the slopes of the [tangent lines](#) to a point on the graph for each  $x$ -value in the domain of  $f(x)$ . For example if one draws a tangent line through the point  $(0.5, 3)$  its slope will be zero. Looking at the graph of  $f'(x)$  at  $x=0.5, y=0$



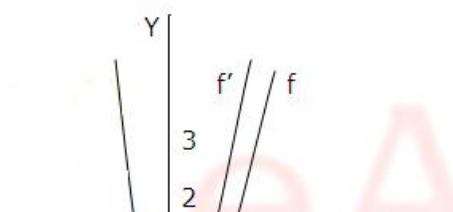
Example: Find the derivative with respect to  $x$  of  $f(x)=x^3-x$ . Graph  $f$  and  $f'$  together and discuss the relationship between the two graphs

Solution:

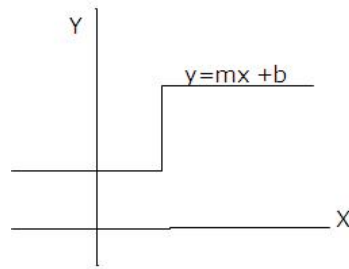
$$f'(x) = \lim_{\omega \rightarrow x} \frac{f(\omega) - f(x)}{\omega - x} = \lim_{\omega \rightarrow x} \frac{(\omega^3 - \omega) - (x^3 - x)}{\omega - x}$$

$$= \lim_{\omega \rightarrow x} \frac{(\omega - x)[(\omega^2 + \omega x + x^2) - 1]}{\omega - x} = \lim_{\omega \rightarrow x} (\omega^2 + \omega x + x^2 - 1)$$

$$= x^2 + x^2 + x^2 - 1 = 3x^2 - 1$$



positive slope, it is negative where the graph of  $f$  has negative slope, and it is zero where the graph of  $f$  is horizontal.



## Interpretation of the derivative

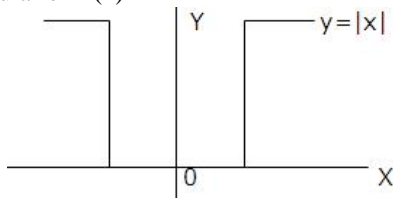
The derivative  $f'$  of a function  $f$  can be interpreted as a function whose value at  $x$  is the slope of the graph of  $y=f(x)$  at  $x$ , or alternatively, it can be interpreted as a function whose value at  $x$  is the instantaneous rate of change of  $y$  with respect to  $x$  at  $x$ . In particular, when  $y=f(t)$  describes the position at time  $t$  of an object moving along a straight line, then  $f'(t)$  describes the instantaneous velocity of the object at time ' $t$ '.

From the figure at each value of ' $x$ ', the tangent line to a line  $y=mx+b$  coincides with the line itself and hence all tangent lines have slope  $m$ . This suggests geometrically that if  $f(x) = mx + b$ , then  $f'(x) = m$  for all  $x$ . This is confirmed by the following computations:

$$\begin{aligned} f'(x) &= \lim_{\omega \rightarrow x} \frac{f(\omega) - f(x)}{\omega - x} = \lim_{\omega \rightarrow x} \frac{(m\omega + b) - (mx + b)}{\omega - x} = \lim_{\omega \rightarrow x} \frac{m\omega - mx}{\omega - x} \\ &= \lim_{\omega \rightarrow x} \frac{m(\omega - x)}{\omega - x} = \lim_{\omega \rightarrow x} m = m \end{aligned}$$

Example: The graph of  $y=|x|$  has a corner at  $x=0$  which implies that  $f(x)=|x|$  is not differentiable at  $x=0$

- Prove that  $f(x)=|x|$  is not differentiable at  $x=0$  by showing that the limit does not exist at  $x=0$
- Find a formula for  $f'(x)$



Solution: From the definition

$$a) \quad f'(0) = \lim_{\omega \rightarrow 0} \frac{f(\omega) - f(0)}{\omega - 0} = \lim_{\omega \rightarrow 0} \frac{|\omega| - |0|}{\omega} = \lim_{\omega \rightarrow 0} \frac{|\omega|}{\omega}$$

$$\text{But, } \frac{|\omega|}{\omega} = \begin{cases} 1, & \omega > 0 \\ -1, & \omega < 0 \end{cases} \text{ so that } \lim_{\omega \rightarrow 0^-} \frac{|\omega|}{\omega} = -1 \text{ and } \lim_{\omega \rightarrow 0^+} \frac{|\omega|}{\omega} = 1$$

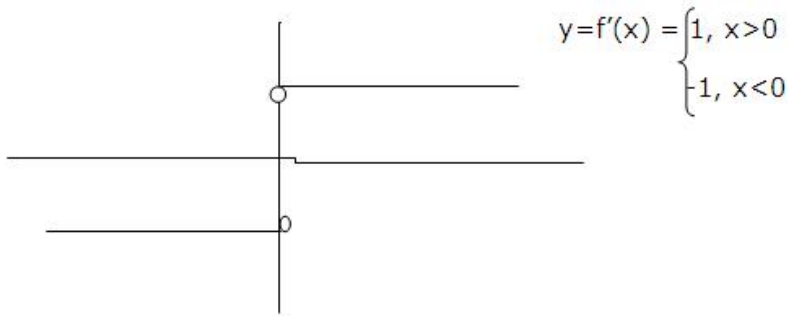
$$\text{Thus } f'(0) = \lim_{\omega \rightarrow 0} \frac{|\omega|}{\omega}$$

does not exist because the one sided limits are not equal.

b) A formula for the derivatives of  $f(x)=|x|$  can be obtained by writing  $|x|$  in piecewise form and treating the cases  $x>0$  and  $x<0$  separately. If  $x>0$ , then  $f(x)=x$  and  $f'(x)=1$ ; if  $x<0$ , then  $f(x)=-x$  and  $f'(x)=-1$ . Thus,

$$f'(x) = \begin{cases} 1, & x > 0 \\ -1, & x < 0 \end{cases}$$

The graph of  $f'$  is shown below. We can see that  $f'$  is not continuous at  $x=0$ . This shows that a function that is continuous everywhere may have a derivative that fails to be continuous everywhere



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## Reference Links:

- <http://en.wikipedia.org/wiki/Derivative>
- [http://en.wikipedia.org/wiki/Difference\\_quotient](http://en.wikipedia.org/wiki/Difference_quotient)
- <http://en.wikipedia.org/wiki/Slope>
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