GRAPHICAL AND ANALYTICAL REPRESENTATION OF DERIVATIVE

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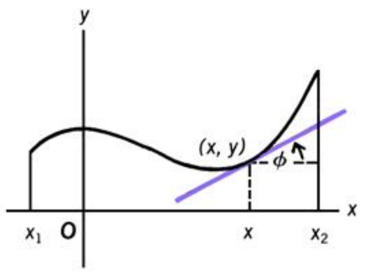
Introduction



The main idea of derivative is that of rate of change of a function. The primary mathematical tool that is used to calculate rate of change and the slopes of curves is derivative. Slope of the graph of y=f(x) at x=x0is given by

$$m_{curve} = \lim_{x_1 \to x_0} \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

The ratio $\frac{f(x_1) - f(x_0)}{x_1 - x_0}$ is called a difference quotient.

The difference quotient can also be interpreted as the average rate of change of f(x) over the interval [x_0, x_1] and its limit as x_1 x_0 is the instantaneous rate of change of f(x) at x=x_0.

Derivative of a function

Suppose that x_0 is a number in the domain of a function f then the derivative of ‘f’ at x=x_0 and is denoted by f’(x_0) and is defined as

$$f'(x_0) = \lim_{x_1 \to x_0} \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

If the limit of the difference quotient exists, f’(x_0) is the slope of the graph of ‘f’ at the point x=x_0. If this limit does not exist, then the slope of the graph of ‘f’ is undefined at x=x_0
Example: Find the slope of the function y=x^2+1 at the point (2, 5)
Solution: Slope of the curve at the point (2, 5) is given by

$$f'(2) = \lim_{x_1 \to 2} \frac{f(x_1) - f(2)}{x_1 - 2} = \lim_{x_1 \to 2} \frac{(x_1^2+1)-5}{x_1 - 2} = \lim_{x_1 \to 2} \frac{x_1^2 - 4}{x_1 - 2}$$

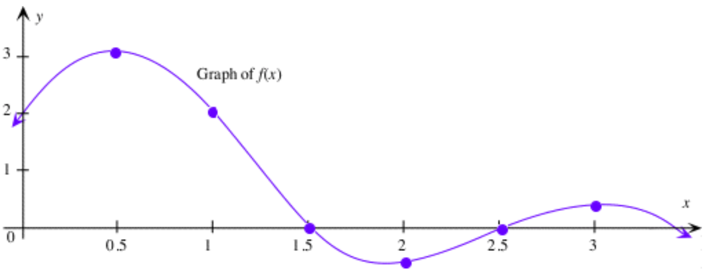
$$= 2 + 2 = 4$$

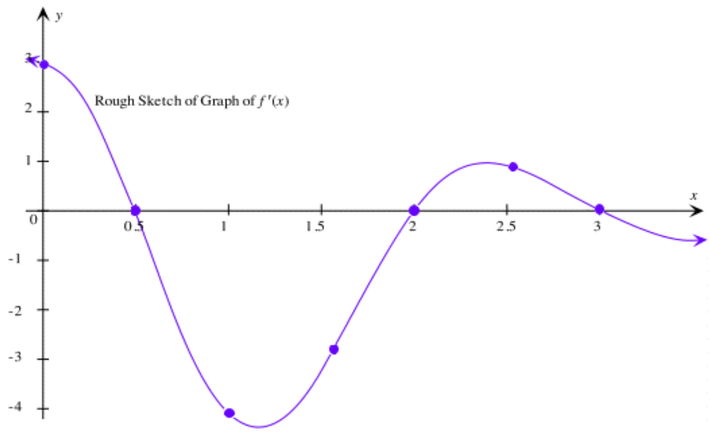
Graphical representation of Derivative

Suppose that x_0 is a number in the domain of a function ‘f’. If

$$f'(x_0) = \lim \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

then we define the tangent line to the graph of f at the point P(x_0, f(x_0)) to be the line whose equation is $y - f(x_0) = f'(x_0)(x - x_0)$. Below is the graph of a function f(x) and a rough sketch of f’(x). The graph of f’(x) represents the slopes of thetangent lines to a point on the graph for each x-value in the domain of f(x). For example if one draws a tangent line through the point (0.5, 3) its slope will be zero. Looking at the graph of f’(x) at x=0.5, y=0





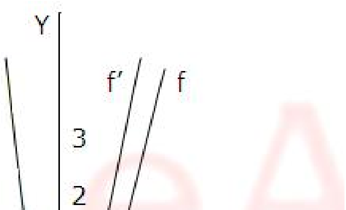
Example: Find the derivative with respect to x of f(x)=x3-x. Graph f and f’ together and discuss the relationship between the two graphs

Solution:

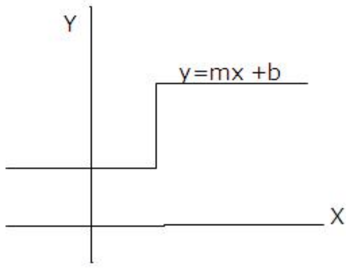
$$f'(x) = \lim_{\omega \to x} \frac{f(\omega) - f(x)}{\omega - x} = \lim_{\omega \to x} \frac{(\omega^3-\omega)-(x^3-x)}{\omega - x}$$

$$= \lim_{\omega \to x} \frac{(\omega-x)[(\omega^2+\omega x+x^2)-1]}{\omega-x} = \lim_{\omega \to x} (\omega^2+\omega x+x^2-1)$$

$$= x^2+x^2+x^2-1 = 3x^2-1$$



positive slope, it is negative where the graph of f has negative slope, and it is zero where the graph of f is horizontal.



Interpretation of the derivative

The derivative f' of a function f can be interpreted as a function whose value at x is the slope of the graph of y=f(x) at x, or alternatively, it can be interpreted as a function whose value at x is the instantaneous rate of change of y with respect to x at x. In particular, when y=f(t) describes the position at time t of an object moving along a straight line, then f'(t) describes the position at time t of an object moving along a straight line, then f'(t) describes the instantaneous velocity of the object at time 't'.

From the figure at each value of 'x', the tangent line to a line y=mx+b coincides with the line itself and hence all tangent lines have slope m. This suggests geometrically that if f(x) = mx + b, then f'(x) = m for all x. This is confirmed by the following computations:

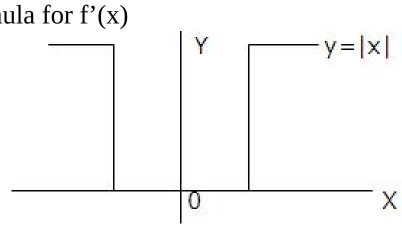
$$f'(x) = \lim_{\omega \to x} \frac{f(\omega) - f(x)}{\omega - x} = \lim_{\omega \to x} \frac{(m\omega+b) - (mx+b)}{\omega - x} = \lim_{\omega \to x} \frac{m\omega - mx}{\omega - x}$$

$$= \lim_{\omega \to x} \frac{m(\omega-x)}{\omega-x} = \lim_{\omega \to x} m = m$$

Example: The graph of y=|x| has a corner at x=0 which implies that f(x)=|x| is not differentiable at x=0

a) Prove that f(x)=|x| is not differentiable at x=0 by showing that the limit does not exist at x=0

b) Find a formula for f'(x)



Solution: From the definition

a) $$f'(0) = \lim_{\omega \to 0} \frac{f(\omega)-f(0)}{\omega - 0} = \lim \frac{|\omega| - |0|}{\omega} = \lim_{\omega \to 0} \frac{|\omega|}{\omega}$$

But, $\frac{|\omega|}{\omega} = \begin{cases} 1, & \omega>0 \\ -1, & \omega<0 \end{cases}$ so that $\lim_{\omega \to 0^-} \frac{|\omega|}{\omega} = -1$ and $\lim_{\omega \to 0^+} \frac{|\omega|}{\omega} = 1$

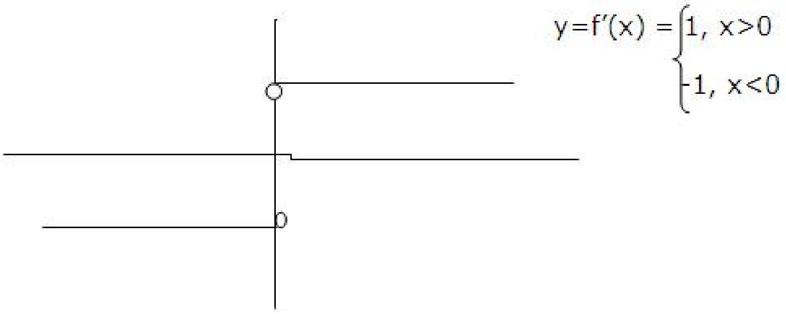
Thus $f'(0) = \lim_{\omega \to 0} \frac{|\omega|}{\omega}$

does not exist because the one sided limits are not equal.

b) A formula for the derivatives of f(x) =|x| can be obtained by writing |x| in piecewise form and treating the cases x>0 and x<0 separately. If x>0, then f(x) =x and f'(x)=1; if x<0, then f(x)-x and f'(x)=-1. Thus,

$$f'(x) = \begin{cases} 1, & x>0 \\ -1, & x<0 \end{cases}$$

The graph of f' is shown below. We can see that f' is not continuous at x=0. This shows that a function that is continuous everywhere may have a derivative that fails to be continuous everywhere



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Reference Links:

- http://en.wikipedia.org/wiki/Derivative
- http://en.wikipedia.org/wiki/Difference_quotient
- http://en.wikipedia.org/wiki/Slope
- http://en.wikipedia.org/wiki/Tangent_lines_to_circles

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