

INDEFINITE INTEGRALS – II

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Integration by Substitution

$$\int f(x) dx.$$

Here the given integral $\int f(x) dx$ can be transformed into another form by changing the [independent variable](#) 'x' to 't' by substituting $x=g(t)$.

Consider $I = \int f(x) dx$

Put $x=g(t)$ so that $dx/dt = g'(t)$

We can write $dx = g'(t)dt$

$$\text{Hence } I = \int f(x) dx = \int f[g(t)] g'(t)dt$$

This change of variable formula is one of the important tools available to us in the name of [integration by substitution](#). Usually, we make a substitution for a function whose derivative also occurs in the integrand.

We can have a glance at few examples.

$$\int \tan x dx = \int \frac{\sin x}{\cos x} dx$$

Put $\cos x = t$, so that $-\sin x dx = dt$

$$i) \int \tan x dx = \int \frac{-dt}{t}$$

$$= -\log|t| + C$$

$$= -\log|\cos x| + C$$

$$= \log|\cos x|^{-1} + C$$

$$= \log|1/\cos x| + C$$

$$= \log|\sec x| + C$$

Hence $\int \tan x dx = \log|\sec x| + C$

Similarly we can find the integral of $\cot x$ also

$\int \cot x dx = \log|\sin x| + C$

$$\int (\log x)^2 dx$$

$$ii) \int \frac{(\log x)^2}{x} dx$$

Here we know the derivative of $\log x$ is $1/x$, so put $\log x = t$

$$\log x = t$$

$$1/x dx = dt$$

$$\int (\log x)^2 dx = \int t^2 dt$$

$$= \frac{t^3}{3} + C$$

$$= \frac{(\log x)^3}{3} + C$$

$$= \frac{(\log x)^3}{3} + C$$

Using substitution technique, we can find the following standard integrals.

- i) $\int \tan x \, dx = \log |\sec x| + C$
- ii) $\int \cot x \, dx = \log |\sin x| + C$
- iii) $\int \sec x \, dx = \log |\sec x + \tan x| + C$
- iv) $\int \operatorname{cosec} x \, dx = \log |\operatorname{cosec} x - \cot x| + C$

Integration using trigonometric identities

When the integrand involves some trigonometric functions, we use some well known identities to find the integrals.

Most commonly used [trigonometric identities](#) are,

- $\cos^2 x = [1 + \cos 2x]/2$
- $\sin^2 x = [1 - \cos 2x]/2$
- $2\cos x \cos y = \cos(x+y) + \cos(x-y)$
- $2\sin x \sin y = \cos(x-y) - \cos(x+y)$
- $2\sin x \cos y = \sin(x+y) + \sin(x-y)$
- $2\cos x \sin y = \sin(x+y) - \sin(x-y)$
- $\sin^3 x = [3\sin x - \sin 3x]/4$
- $\cos^3 x = [3\cos x + \cos 3x]/4$

Integrals of special form

While integrating functions of the form $\frac{\int px+q \, dx}{ax^2+bx+c}$ and $\frac{\int px+q \, dx}{\sqrt{ax^2+bx+c}}$

where p, q, a, b and c are constants we have to use another method.

Assume that $\frac{px+q}{ax^2+bx+c} = \frac{A}{d} \frac{d(ax^2+bx+c)}{dx} + \frac{B}{d}$

Which means that Numerator = A derivative $(ax^2+bx+c) + B$

To determine A and B, we equate from both sides the coefficients of 'x' and constant terms. After getting the values of A and B we can reduce the integral to one of the known forms.

Integration by Parts

[Integration by Parts](#) is applicable only for the functions which are expressed as product of functions.

If 'u' and 'v' are any two differentiable functions of a single variable 'x', then

$$\int uv \, dx = u \int v \, dx - \int [u' \int v \, dx] \, dx$$

OR

$$\int f(x) g(x) \, dx = f(x) \int g(x) \, dx - \int [f'(x) \int g(x) \, dx] \, dx.$$

This formula can be stated as follows: "The integral of the product of two functions = (first function) x (integral of the second function) – Integral of [(derivative of first function) x (integral of the second function)]".

Example: Find $\int x \sin 3x \, dx$

Solution: $\int x \sin 3x \, dx = x \int \sin 3x \, dx - \int [d(x) \int \sin 3x \, dx] \, dx$

$$= x \frac{-\cos 3x}{3} - \int 1 \frac{-\cos 3x}{3} \, dx$$

$$= \frac{-x \cos 3x}{3} + \frac{\sin 3x}{9} + C$$

Note: While performing Integration by Parts we have memorize “ILATE” which gives the order in which the functions are to be taken. Inverse and [logarithmic functions](#) must be taken as first function since they doesn't have integrals.

I = Inverse function

L = Logarithmic function

A = Algebraic Function

T = Trigonometric function

E = Exponential function

Integrals of the type $\int e^x [f(x) + f'(x)] dx$

$$\begin{aligned} \text{We have } I &= \int e^x [f(x) + f'(x)] dx \\ &= \int e^x f(x) dx + \int e^x f'(x) dx \\ &= I_1 + \int e^x f'(x) dx \text{ where } I_1 = \int e^x f(x) dx \dots\dots(1) \end{aligned}$$

$$\begin{aligned} I_1 &= \int e^x f(x) dx \\ &= \int f(x) e^x dx \quad [\text{Take } f(x) \text{ as 1st function}] \\ &= f(x) e^x - \int f'(x) e^x dx \\ &= f(x) e^x - \int f'(x) e^x dx + C \end{aligned}$$

$$\begin{aligned} (1) \text{ becomes, } I &= \int e^x f(x) - \int f'(x) dx + \int f'(x) dx + C \\ &= \int e^x f(x) + C \end{aligned}$$

$$\text{Hence } \int e^x [f(x) + f'(x)] dx = \int e^x f(x) + C$$

Example: Find $\int e^x [\sin x + \cos x] dx$

Solution: $\int e^x [\sin x + \cos x] dx = \int e^x \sin x + C$ [If $f(x) = \sin x$ then $f'(x) = \cos x$]

Integrals of some more types

Here we discuss some special types of standard integrals based on the technique of integration by parts:

$$i) \int \sqrt{x^2 - a^2} dx = \frac{x\sqrt{x^2 - a^2}}{2} - \frac{a^2 \log |x + \sqrt{x^2 - a^2}|}{2} + C$$

$$ii) \int \sqrt{x^2 + a^2} dx = \frac{x\sqrt{x^2 + a^2}}{2} + \frac{a^2 \log |x + \sqrt{x^2 + a^2}|}{2} + C$$

$$iii) \int \sqrt{a^2 - x^2} dx = \frac{x\sqrt{a^2 - x^2}}{2} + \frac{a^2 \sin^{-1} \left(\frac{x}{a} \right)}{2} + C$$

Now try it yourself! Should you still need any help, [click here](#) to schedule live online session with e Tutor!

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Reference Links:

- http://en.wikipedia.org/wiki/Integration_by_parts
<http://www.analyzmath.com/logfunction/logfunction.html>
- http://en.wikipedia.org/wiki/Integration_by_substitution
- http://en.wikipedia.org/wiki/List_of_trigonometric_identities
- http://en.wikipedia.org/wiki/Dependent_and_independent_variables

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