INDEFINITE INTEGRALS – II

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Integration by Substitution

$$\int f(x)\,dx.$$

Here the given integral ?f(x) dx can be transformed into another form by changing the independent variable 'x' to 't' by substituting x=g(t).

Consider I = ?f(x) dx
Put x=g(t) so that dx/dt = g'(t)
We can write dx = g'(t)dt
Hence I = ?f(x) dx = ?f[g(t)] g'(t)dt
This change of variable formula is one of the important tools available to us in the name of integration by substitution. Usually, we make a substitution for a function whose derivative also occurs in the integrand.
We can have a glance at few examples.

i) $\int \tan x\,dx = \int \frac{\sin x\,dx}{\cos x}$

Put cosx = t, so that –sinxdx=dt

$\int \tan x\,dx = \int \frac{-dt}{t}$

= - log|t| + C
= -log|cosx| + C
= $\log|\cos x|^{-1}$ + C
= log|1/cosx| + C
= log|secx| + C

Hence **?tanx dx = log|secx| + C**
Similarly we can find the integral of cotx also
?cotx dx = log|sinx| + C

ii) $\int \frac{(\log x)^2\,dx}{x}$

Here we know the derivative of logx is 1/x, so put logx = t
log x = t
1/x dx = dt

$\int \frac{(\log x)^2\,dx}{x}$ = $?t^2$ dt

= $t^3/3$ + C

= $\frac{(\log x)^3}{3}$ + C

Using substitution technique, we can find the following standard integrals.

i) ?tanx dx = log |secx| + C

ii) ?cotx dx = log |sinx| + C

iii) ?secx dx = log |secx + tanx| + C

iv) ?cosecx dx = log | cosecx - cotx| + C

Integration using trigonometric identities

When the integrand involves some trigonometric functions, we use some well known identities to find the integrals.

Most commonly used trigonometric identities are,

- $Cos^2x = [1+cos2x]/2$
- $Sin^2x = [1-cos2x]/2$
- 2cosxcosy= cos(x+y) + cos(x-y)
- 2sinxsiny = cos(x-y) – cos(x+y)
- 2sinxcosy = sin(x+y) + sin(x-y)
- 2cosxsiny = sin(x+y) – sin(x-y)
- $Sin^3x = [3sinx - sin3x]/4$
- $Cos^3x = [3cosx + cos3x]/4$

Integrals of special form

While integrating functions of the form $\int \frac{px+q}{ax^2+bx+c} dx$ and $\int \frac{px+q}{\sqrt{ax^2+bx+c}} dx$

where p, q, a, b and c are constants we have to use another method.

Assume that $px+q = A \frac{d}{dx} (ax^2+bx+c) + B = A(2ax+b) + B$

Which means that Numerator = A derivative (ax^2+bx+c) + B

To determine A and B, we equate from both sides the coefficients of ‘x’ and constant terms. After getting the values of A and B we can reduce the integral to one of the known forms.

Integration by Parts

Integration by Parts is applicable only for the functions which are expressed as product of functions.

If ‘u’ and ‘v’ are any two differentiable functions of a single variable ‘x’, then

?uv dx = u?vdx - ?[u’?vdx] dx

OR

?f(x) g(x)dx = f(x) ?g(x)dx - ?[f’(x)?g(x)dx]dx.

This formula can stated as follows: “The integral of the product of two functions = (first function) x (integral of the second function) – Integral of [(derivative of first function) x (integral of the second function)]”.

Example: Find ?x sin3x dx

Solution: ?x sin3x dx = x ?sin3xdx - ?[d(x)/dx ?sin3xdx] dx

= x (-cos3x)/3 - ?1 (–cos3x)/3 dx

$$= \frac{-xcos3x}{3} + \frac{sin3x}{9} + C$$

Note: While performing Integration by Parts we have memorize "ILATE" which gives the order in which the functions are to be taken. Inverse and logarithmic functions must be taken as first function since they doesn't have integrals.

I = Inverse function

L = Logarithmic function

A = Algebraic Function

T = Trigonometric function

E = Exponential function

Integrals of the type ?e^x [f(x) + f'(x)] dx

We have I=?e^x [f(x) + f'(x)]dx

= ?e^x f(x)dx + ?e^xf'(x)dx

= I1 + ?e^x f'(x) dx where I1=?e^x f(x) dx ……..(1)

I1 = ?e^x f(x) dx

= ?f(x) e^x dx [Take f(x) as 1st function]

= f(x) e^x - ?f'(x) e^x dx

= f(x) e^x - ?f'(x) e^x dx + C

(1) becomes, I = e^x f(x) - ?f'(x) dx + ?f'(x) dx + C

= e^x f(x) + C

Hence ?e^x [f(x) + f'(x)] dx = e^x f(x) + C

Example: Find ?e^x[sinx + cosx] dx

Solution: ?e^x[sinx + cosx] dx = e^x sinx + C [If f(x)=sinx then f'(x)=cosx]

Integrals of some more types

Here we discuss some special types of standard integrals based on the technique of integration by parts:

i) $\int\sqrt{x^2-a^2}\,dx = \frac{x\sqrt{x^2-a^2}}{2} - \frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right| + C$

ii) $\int\sqrt{x^2+a^2}\,dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\log\left|x+\sqrt{x^2+a^2}\right| + C$

iii) $\int\sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + C$

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Reference Links:

- http://en.wikipedia.org/wiki/Integration_by_parts
  http://www.analyzemath.com/logfunction/logfunction.html
- http://en.wikipedia.org/wiki/Integration_by_substitution
- http://en.wikipedia.org/wiki/List_of_trigonometric_identities
- http://en.wikipedia.org/wiki/Dependent_and_independent_variables

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