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INSTANTANEOUS RATE OF CHANGE AS A LIMIT OF AVERAGE RATE OF CHANGE

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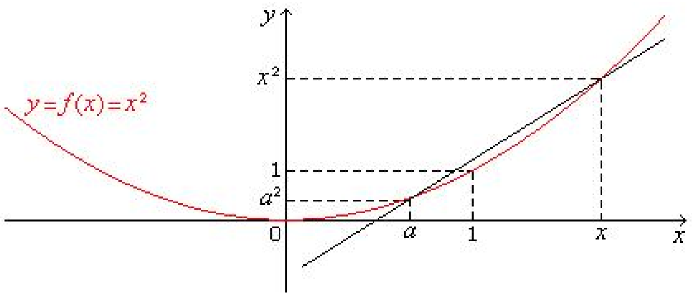
Introduction



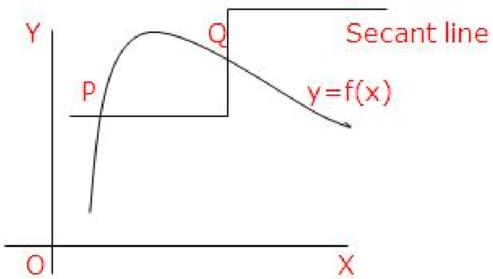
In this topic we will discuss about real-world phenomena involving change in quantities – speed of a rocket, inflation of currency, number of bacteria in a culture etc. Here we will interpret both average and instantaneous velocity geometrically and we will define the slope of curve at a point. In this section we will explore the connection between velocity at an instant, the slope of a curve at a point and rate of change.

If an object moves along an s-axis and if the position versus time curve is s=f(t), then the average velocity of the object between times t_0 and t_1

$$v_{ave} = \frac{f(t_1) - f(t_0)}{t_1 - t_0}$$

It is represented geometrically by the slope of the line joining the points $(t_0, f(t_0))$ and $(t_1, f(t_1))$.

Slope of a secant line



Consider the function y=f(x) whose graph is shown in the figure. The line through two points on the curve is called a secant line. Here PQ is a secant line. We know to calculate the slope of a line through two points. Let $P(x_0, f(x_0))$ and $Q(x_1, f(x_1))$ be the given points then slope of the secant line PQ is given by

$$m_{sec} = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

As the sampling point $Q(x_1, f(x_1))$ is chosen closer to P, that is, as x_1 is selected closer to x_0, the slopes msec more nearly approximate what might reasonably call the slope of the curve y = f(x) at the point P. Thus from the above equation slope of the curve y=f(x) at $P(x_0, f(x_0))$ is defined by

$$m_{curve} = \lim_{x_1 \to x_0} \frac{f(x_1)-f(x_0)}{x_1 - x_0}$$

Example: Consider the function f(x) = 6x-x2 and the point P(2, f(2)) = (2, 8).

a) Find the slopes of secant lines to the graph of y=f(x) determined by P and points on the graph at x=3 and x=1.5

b) Find the slope of the graph of y=f(x) at the point P.
Solution:

a) The secant line to the graph of ‘f’ through P and Q(3, f(3)) = (3, 9) has slope

$$m_{sec} = \frac{9-8}{3-2} = 1$$

The secant line to the graph of ‘f’ through P and Q(1.5, f(1.5)) =(1.5, 6.75) has the slope

$$m_{sec} = \frac{6.75 - 8}{1.5 - 2} = 2.5$$

b) The slope of the graph of f at the point P is

$$m_{curve} = \lim_{x_1 \to 2} \frac{f(x_1) - f(2)}{x_1 - 2} = \lim_{x_1 \to 2} \frac{6x_1 - x_1^2 - 8}{x_1 - 2}$$

$$= \lim_{x_1 \to 2} \frac{(4-x_1)(x_1-2)}{x_1 - 2}$$

$$= \lim_{x_1 \to 2} (4 - x_1)$$

$$= 4 - 2 = 2$$

Geometrical interpretation of instantaneous velocity

If a particle moves along an s-axis, and if the position versus time curve is s =f(t), then the instantaneous velocity of the particle at time t0,

$$v_{inst} = \lim_{t_1 \to t_0} \frac{f(t_1) - f(t_0)}{t_1 - t_0}$$

It is represented geometrically by the slope of the curve at the point (t0, f(t0)).
In general, if x and y are quantities related by an equation y=f(x), we can consider the rate at which y changes with x. As with velocity, we distinguish between an average rate of change, represented by the slope of secant line to the graph of y=f(x), and an instantaneous rate of change, represented by the slope of the curve at a point.

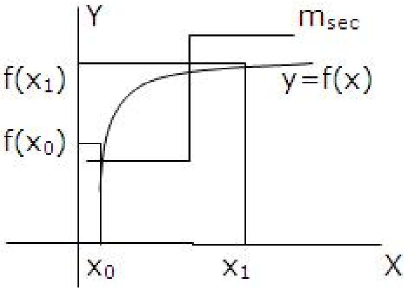
Slopes and rates of change

If y=f(x), then the average rate of change of 'y' with respect to 'x' over the interval [x_0, x_1] is

$$r_{ave} = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

Geometrically, the average rate of change of y with respect to x over the interval [x_0, x_1] is the slope of the secant line to the graph of y=f(x) through the points (x_0, f(x_0)) and (x_1, f(x_1))

$$r_{ave} = m_{sec}$$

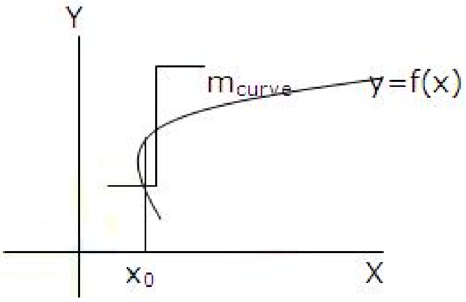


If y=f(x), then the instantaneous rate of change of 'y' with respect to 'x' when x=x_0 is

$$r_{inst} = \lim_{x_1 \to x_0} \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

Geometrically, the instantaneous rate of change of y with respect to x when x=x_0 is the slope of the graph of y=f(x) at the point (x_0, f(x_0)).

$$r_{inst} = m_{curve}$$



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Example: Let y=x^2+1

a) Find the average rate of change of y with respect to x over the interval [3, 5]

b) Find the instantaneous rate of change of y with respect to x when x=-4

Solution: a) Given f(x) = x^2+1, x_0=3 and x_1=5

$$r_{ave} = \frac{f(x_1) - f(x_0)}{x_1 - x_0} = \frac{f(5) - f(3)}{5 - 3} = \frac{26-10}{2} = 8$$

Thus, on an average, y increases 8 units per unit increase in x over the interval [3, 5].

$$r_{inst} = \lim_{x_1 \to x_0} \frac{f(x_1) - f(x_0)}{x_1 - x_0} = \lim_{x_1 \to x_0} \frac{f(x_1) - f(-4)}{x_1-(-4)}$$

$$= \lim_{x_1 \to -4} \frac{(x_1^2+1) - 17}{x_1+4}$$

$$= \lim_{x_1 \to -4} \frac{x_1^2 -16}{x_1+4}$$

$$= \lim_{x_1 \to -4} \frac{(x_1+4)(x_1-4)}{x_1+4}$$

Thus, for a small change in x from x=-4, the value of y will change approximately eight times as much in the opposite direction, since the instantaneous is negative, the value of y decreases as values of x move through x=-4 from left to right.

Rates of change in Applications

In applied problems, average rate of change must be accompanied by appropriate units. In general, the units for a rate of change of y with respect to x are obtained by “dividing” the units of y by the units of x and then simplifying according to the standard rules of algebra. Here are some examples:

a) If y is in degrees Fahrenheit (?F) and x is in inches)in), then a rate of change of y with respect to x has units of degrees Fahrenheit per inch (?F/in)

b) If y is in feet per second (ft/s) and x is in seconds(s), then a rate of change of y with respect to x has units of feet per second per second (ft/s/s), which would usually be written as ft/s^2

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Reference Links

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- http://en.wikipedia.org/wiki/Secant_line
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