INTRODUCTION TO INTEGRATION

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Integration

$$\int f(x)\,dx.$$

Calculus is centered on the concept of the derivative. If a function 'f' is differentiable in an interval I, then its derivative f' exists at each point of I. We can think about the question, if f' exists at each point of I then what will be the function? Such functions are called anti-derivatives. The formula that gives all these anti-derivatives is called the indefinite integral of the function and the process of finding anti-derivatives is called integration. The development of integral calculus arises out of the efforts of solving the problems of the following types:

i) The problem of finding a function whenever its derivative is given.

ii) The problem of finding the area bounded by the graph of a function under certain conditions.

These two types of problems lead to indefinite and definite integrals, which together constitute the Integral Calculus.

Integration as an Inverse Process of Differentiation

Integration is the inverse process of differentiation. If we are given the derivative of a function and asked to find its primitive (original function), the process is called integration or anti-differentiation.

Let us consider few examples,

i) $\frac{d}{dx}(\tan x) = \sec^2 x$ …………….(1)

The function $\sec^2 x$ is the derived function of tanx, so we say that tanx is the anti-derivative of $\sec^2 x$

ii) $\frac{d}{dx}(\log x) = \frac{1}{x}$ ……………..(2)

The function 1/x is the derived function of logx, so we that logx is the anti-derivative of 1/x.

We know that the derivative of any real number C, treated as a Constant Function is zero. And hence we can write (1) and (2) as

$\frac{d}{dx}(\tan x + C) = \sec^2 x$ $\frac{d}{dx}(\log x + C) = \frac{1}{x}$

Hence the anti-derivatives (integrals) given above are not unique. There exists infinitely many integrals of each these functions which can be obtained by choosing C arbitrarily from the set of real numbers. Hence C is referred to as arbitrary constant.

In general, if there is a function F such that $\frac{d}{dx} F(x) = f(x)$ for all x?I then for any arbitrary real number C,

$\frac{d}{dx}[F(x)+C] = f(x)$, x ? I

We introduce the new symbol, ?f(x) dx which will represent the entire class of anti derivatives read as the indefinite integral of ‘f’ w. r. t x.

Symbolically, we write ?f(x) dx = F(x) + C

f(x) in ?f(x)dx is the integrand, ‘x’ in f(x) is the variable of integration, ‘C’ is the constant of integration.

Geometrical interpretation of indefinite integral

The statement ?f(x) dx=F(x) + C=y, represents a family of curves. The different values of C will correspond to different members of this family and these members can be obtained by shifting any one of the curves parallel to itself. This is the geometrical interpretation of indefinite integral.

Some properties of indefinite integral

1) The process of differentiation and integration are inverses of each other which results in

?f’(x)dx=f(x)+C, where C is any arbitrary constant.

2) Two indefinite integrals with the same derivative lead to the same family of curves and so they are equivalent.

Let ‘f’ and ‘g’ be two functions such that

$\frac{d}{dx}$?f(x) dx= $\frac{d}{dx}$?g(x) dx

3) ?[f(x)+g(x)]dx = ?f(x)dx + ?g(x)dx

4) For any real number ‘k’, ?k f(x) dx= k ?f(x) dx

5) Properties 3 and 4 can be generalized to a finite number of functions f1, f2 ……….. fn and the real numbers k_1, k_2…………k_n giving

? [$k_1 f_1(x) + k_2 f_2(x)$ +…..+ $k_n f_n(x)$] dx = k_1?$f_1(x)$dx + k_2?$f_2(x)$dx + …….+kn?fn(x)dx.

Comparison between differentiation and integration

1. Both are operations on functions.
2. Both satisfy the property of linearity. For any two constants k_1 and k_2,

a) $\frac{d}{dx}[k_1f_1(x)+k_2f_2(x)] = k_1\frac{d}{dx}[f_1(x)] + k_2\frac{d}{dx}[f_2(x)]$

b) $?[k_1f_1(x)+k_2f_2(x)]\,dx = k_1?f_1(x)\,dx + k_2?f_2(x)dx$

3. All the functions are not differentiable, in the same way all the functions are not integrable.
4. The derivative of a function if it exists, is unique, but the integral of a function is not unique.
5. The derivative of a polynomial function is a polynomial function with power less than one, but the integral of a polynomial function is a polynomial function with power more than one.
6. We can find derivative at a point, but the integral of a function exist only for an interval not at a point.
7. The geometrical meaning of a derivative is that it is the slope of tangent to a corresponding curve at a point but the geometrical meaning of an indefinite integral is a family of curves placed parallel to each other having parallel tangents at the points of intersection of the curves of the family with the lines orthogonal to the axis representing the variable of integration.

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Reference Links:

- http://en.wikipedia.org/wiki/Indefinite_integrals
- http://en.wikipedia.org/wiki/Calculus
- http://www.elainetron.com/apcalc/topic4.htm
- http://en.wikipedia.org/wiki/Definite_Integrals
- http://en.wikipedia.org/wiki/Rational_function_modeling

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