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# LIMITS

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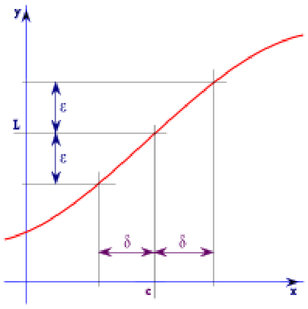
# Introduction to Limits




The word limit in this topic represents the limiting value of a function at a given point. In order to learn continuity and differentiability, we must be familiar with the topic 'limits'. Here we learn some standard results in limits, algebra of limits, theorems related to limits etc.
Consider the function f(x) = $3x^2$. We can see that as 'x' takes the value very close to 0, the value of f(x) also moves towards 0, so we say

$$\lim_{x \to 0} f(x) = 0$$

We will read this as limit x tends to zero f(x) equals zero
In general, as $x \to a,\ f(x) \to l,$ then 'l' is called limit of the function f(x) which is symbolically written as $\lim_{x \to a} f(x) = l$
In some cases the function f(x) takes different values to the left of 'a' and right of 'a' so we have to find the left hand limit as well as right hand limit.
We say $\lim_{x \to 0} f(x)$ is the expected value of 'f' at x=a given the values of f near

'x' to the left of a. This value is called the left hand limit of f at 'a'
Similarly $\lim_{x \to 0} f(x)$ is the expected valu of 'f' at x=a given the values of f near

'x' to the right of 'a'. This value is called the right hand limits of 'f' at 'a'.
If the right and left hand limits coincide, we call that common value as the limit of f(x) at x=a and denote it by $\lim_{x \to a} f(x)$.

Example: Example: Evaluate $\lim_{r \to 1} \pi r^2$

Solution: $\lim_{r \to 1} \pi r^2 = \pi(1)^2 = \pi \times 1 = \pi$

# Algebra of limits

Let 'f' and 'g' be two functions such that both $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ exist.

then the following are algebra of limits.

1) Limit of sum of two functions is the sum of the limits of the functions.

$$\lim_{x\to a} [f(x) + g(x)] = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$$

2) Limit of difference of two functions is difference of the limits of the functions.

$$\lim_{x\to a} [f(x) - g(x)] = \lim_{x\to a} f(x) - \lim_{x\to a} g(x)$$

1.

3) Limit of the product of two functions is product of the limits of the functions.

$$\lim_{x\to a} [f(x). g(x)] = \lim_{x\to a} f(x) . \lim_{x\to a} g(x)$$

4) Limit of quotient of two functions is quotient of the limits of the function [provided denominator is non-zero].

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$$

# Limits of polynomials and rational functions

A function f is said to be a polynomial function if f(x) is zero function of if f(x) $=a_0+a_1x+a_2x^2+\dots\dots\dots+a_nx^n$, where $a_i$ 's are real numbers such that $a_n$?0 for some natural number n.

$$\lim_{x\to a} f(x) = \lim_{x\to a} [a_0+a_1x+a_2x^2+a_3x^3+\dots\dots+a_nx^n]$$

$$= \lim_{x\to a} a_0 + \lim_{x\to a} a_1x + \lim_{x\to a} a_2x^2 + \dots\dots\lim_{x\to a} a_nx^n$$

$$= a_0 + a_1\lim_{x\to a} x + a_2\lim_{x\to a} x^2+\dots\dots\dots\dots+a_n\lim_{x\to a} x^n$$

$$= a_0 + a_1 a + a_2 a^2 + \dots\dots\dots\dots + a_n a^n$$

We have, $= f(a)$

Hence $\lim_{x\to a} f(x) = f(a)$

A function is said to be a rational function, if $f(x) = \frac{g(x)}{h(x)}$, where g(x) and h(x) are polynomials such that h(x)?0

2.

$$\lim_{x\to a} f(x) = \lim_{x\to a} \frac{g(x)}{h(x)} = \frac{\lim_{x\to a} g(x)}{\lim_{x\to a} h(x)} = \frac{g(a)}{h(a)}$$

Example: Find the limits:

1) $\lim_{x\to 4} \frac{4x+3}{x-2}$

$$\lim_{x\to 4} \frac{4x+3}{x-2} = \frac{4 \times 4 + 3}{4 - 2} = \frac{19}{2}$$

# Limits of Trigonometric Functions

The following stated theorems about functions in general come in handy in calculating limits of some trigonometric functions.

- Let f and g be two real valued functions with the same domain such that f(x) ? g(x) for all x in the domain of definitions, for some 'a', if both

3. $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ exist, then $\lim_{x \to a} f(x) \leq \lim_{x \to a} g(x)$

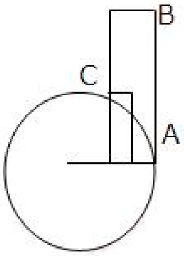
- Sandwich Theorem: Let f, g and h be real functions such that f(x)?g(x)?h(x) for all x in the common domain of definition. For some real number 'a' , if

$$\lim_{x \to a} f(x) = l = \lim_{x \to a} h(x), \text{ then } \lim_{x \to a} g(x) = l$$

The geometric proof of an important inequality relating trigonometric functions is given below.

$$\cos x < \frac{\sin x}{x} < 1 \text{ for } 0<|x|<\pi/2$$

We know that sin(-x) = -sinx and cos(-x) = cosx. Hence, it is sufficient to prove the inequality for 0<x<?/2
"O" is the center of the unit circle such that angle AOC is 'x' radians and 0<x<?/2. Line segments BA and CD are perpendiculars to OA. Join AC. Then,




Area of ?OAC < Area of sector OAC < Area of ?OAB

½ OA.CD < (x/2?). ?(OA)$^2$ < ½ OA.OB
CD < x . OA < AB
From ?OCD,
Sinx = CD/OA (since CD=OA) and hence CD=OA Sinx. Also tanx = AB/OA and therefore, AB = OA. Tanx. Thus, OA. sinx < OA. X < OA. tanx
Since OA is the length, it is positive, we have
sinx < x < tanx
Since 0<x<?/2, sinx is positive and dividing throughout by sinx, we have
1 < x/sinx < 1/cosx
Taking reciprocals, $\cos x < \frac{\sin x}{x} < 1$

4.

# Some Standard Results in Limits

- $\lim_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1}$
- $\lim_{x \to 0} \frac{\sin x}{x} = 1$
- $\lim_{x \to 0} \frac{1 - \cos x}{x} = 0$

Example: Evaluate $\lim_{x \to 0} \frac{\sin ax}{\sin bx}$

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## Reference Links:


- http://en.wikipedia.org/wiki/Limit_%28mathematics%29
- http://en.wikipedia.org/wiki/Limit_%28mathematics%29
- http://en.wikipedia.org/wiki/Polynomial
- http://en.wikipedia.org/wiki/Squeeze_theorem
- http://www.iwu.edu/~lstout/limitTheorems/node3.html


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