MAXIMA AND MINIMA (2nd DERIVATIVE TEST)

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Second derivative test

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maximaandminima2nd derivative test1

Let ‘f’ be a function defined on an interval I and c ? I. Let ‘f’ be twice differentiable at ‘c’. Then

- x = c is a point of local maxima if f’(c) = 0 and f”(c) < 0. The value   f(c) is local maximum value of ‘f’.
- x = c is a point of local minima if f’(c) = 0 and f”(c) > 0. The value   f(c) is local minimum value of ‘f’.
- The test fails, if f’(c) = 0 and f”(c) = 0. In this case, we have to go for1st derivative test and find whether ‘c’ is a point of local maxima, local minima or a point of inflection.

Let’s understand the concept with the help of following example:
Find the local maximum and local minimum of the function:

$f(x) = 3x^4 - 4x^3 -12x^2 + 12$

We have $f(x) = 3x^4 - 4x^3 - 12x^2 + 12$

$f'(x) = 12x^3 - 12x^2 - 24x$

$f'(x) = 12x (x - 1)(x + 2)$

$f''(x) = 36x^2 + 24x – 24 = 12(3x^2 + 2x - 1)$

$f''(0) = -12 < 0$

$f''(1) = 48 > 0$

$f''(-2)= 84 > 0$

Hence bysecond derivative test, x = 0 is a point of local maxima and local maximum value is f(0) = 12, while x = 1 and x = -2 are the points of local minima and local minimum values of ‘f’ are 7 and -20 respectively.

Maximum and minimum values of a function in a closed interval

Let f be a continuous function of an interval I = [a, b]. The f has absolute minimum value and absolute maximum value in I.

Working Rule:

Step I: Find all critical points of ‘f’ in the interval

Step II: Take the end points of the interval

Step III: At all these points calculate the values of ‘f’

Step IV: Identify the maximum and minimum values of ‘f’ out of the values calculated in Step 3. The maximum will be the absolute maximum value of f and the minimum value will be the absolute minimum value of f.

Let’s understand the concept with the help of following example:

Find the absolute maximum and minimum values of a function f given by $f(x) = 2x^3 - 15x^2 + 36x + 1$ on the interval [1, 5]

Given $f(x) = 2x^3 - 15x^2 + 36x + 1$

$f'(x) = 6x^2 - 30x + 36$

$= 6(x - 3)(x - 2)$

$f'(x) = 0$ gives x = 2, 3

f(1) = 24

f(2) = 29

f(3) = 28

f(5) = 56

Hence absolute maximum value of f is 56 at x = 5 and absolute minimum value of f is 24 at x = 1.

Now try it yourself! Should you still need any help, click here to schedule live online session with e Tutor!

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Reference Links:

- http://en.wikipedia.org/wiki/Second_derivative_test
- http://en.wikipedia.org/wiki/Differentiable_function
- http://mathworld.wolfram.com/LocalMaximum.html
- http://mathworld.wolfram.com/LocalMinimum.html
- http://en.wikipedia.org/wiki/First_derivative_test
- http://en.wikipedia.org/wiki/Second_derivative_test

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