

# MAXIMA AND MINIMA (2nd DERIVATIVE TEST)

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## Second derivative test

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maximaandminima2nd derivative test1.gif

Let 'f' be a function defined on an interval I and  $c \in I$ . Let 'f' be twice [differentiable](#) at 'c'. Then

- $x = c$  is a point of local maxima if  $f'(c) = 0$  and  $f''(c) < 0$ . The value  $f(c)$  is [local maximum](#) value of 'f'.
- $x = c$  is a point of local minima if  $f'(c) = 0$  and  $f''(c) > 0$ . The value  $f(c)$  is [local minimum](#) value of 'f'.
- The test fails, if  $f'(c) = 0$  and  $f''(c) = 0$ . In this case, we have to go for [1st derivative test](#) and find whether 'c' is a point of local maxima, local minima or a point of inflection.

Let's understand the concept with the help of following example:

Find the local maximum and local minimum of the function:

$$f(x) = 3x^4 - 4x^3 - 12x^2 + 12$$

$$\text{We have } f(x) = 3x^4 - 4x^3 - 12x^2 + 12$$

$$f'(x) = 12x^3 - 12x^2 - 24x$$

$$f'(x) = 12x(x-1)(x+2)$$

$$f''(x) = 36x^2 + 24x - 24 = 12(3x^2 + 2x - 1)$$

$$f''(0) = -12 < 0$$

$$f''(1) = 48 > 0$$

$$f''(-2) = 84 > 0$$

Hence by [second derivative test](#),  $x = 0$  is a point of local maxima and local maximum value is  $f(0) = 12$ , while  $x = 1$  and  $x = -2$  are the points of local minima and local minimum values of 'f' are 7 and -20 respectively.

## Maximum and minimum values of a function in a closed interval

Let  $f$  be a continuous function of an interval  $I = [a, b]$ . The  $f$  has absolute minimum value and absolute maximum value in  $I$ .

**Working Rule:**

Step I: Find all critical points of ' $f$ ' in the interval

Step II: Take the end points of the interval

Step III: At all these points calculate the values of ' $f$ '

Step IV: Identify the maximum and minimum values of ' $f$ ' out of the values calculated in Step 3. The maximum will be the absolute maximum value of  $f$  and the minimum value will be the absolute minimum value of  $f$ .

Let's understand the concept with the help of following example:

Find the absolute maximum and minimum values of a function  $f$  given by  $f(x) = 2x^3 - 15x^2 + 36x + 1$  on the interval  $[1, 5]$

$$\text{Given } f(x) = 2x^3 - 15x^2 + 36x + 1$$

$$f'(x) = 6x^2 - 30x + 36$$

$$= 6(x - 3)(x - 2)$$

$$f'(x) = 0 \text{ gives } x = 2, 3$$

$$f(1) = 24$$

$$f(2) = 29$$

$$f(3) = 28$$

$$f(5) = 56$$

Hence absolute maximum value of  $f$  is 56 at  $x = 5$  and absolute minimum value of  $f$  is 24 at  $x = 1$ .

Now try it yourself! Should you still need any help, [click here](#) to schedule live online session with e Tutor!

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## Reference Links:

- [http://en.wikipedia.org/wiki/Second\\_derivative\\_test](http://en.wikipedia.org/wiki/Second_derivative_test)
- [http://en.wikipedia.org/wiki/Differentiable\\_function](http://en.wikipedia.org/wiki/Differentiable_function)
- <http://mathworld.wolfram.com/LocalMaximum.html>
- <http://mathworld.wolfram.com/LocalMinimum.html>
- [http://en.wikipedia.org/wiki/First\\_derivative\\_test](http://en.wikipedia.org/wiki/First_derivative_test)
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