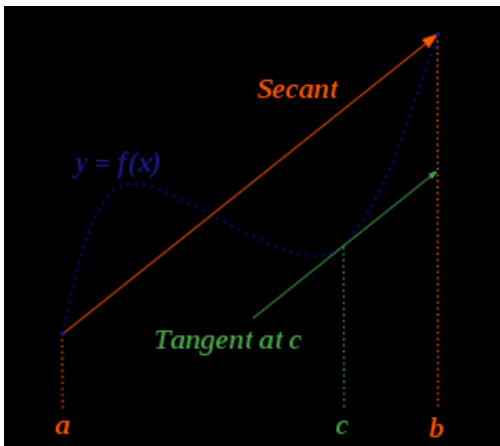


# Mean Value Theorem

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# Rolle's Theorem



Let  $f: [a, b] \rightarrow \mathbb{R}$  be continuous on  $[a, b]$  and differentiable in  $(a, b)$ , such that  $f(a) = f(b)$ , where 'a' and 'b' are some real numbers, then there exists some 'c' in  $(a, b)$  such that  $f'(c) = 0$

For example: Verify [Rolle's theorem](#) for the function  $f(x) = x^2 + 2x - 8$ ,  $x \in [-4, 2]$ .

Solution: Since  $f(x)$  is a [polynomial function](#), it is continuous in  $[-4, 2]$ ,

$f'(x) = 2x + 2$  is differentiable in  $(-4, 2)$

$$f(-4) = (-4)^2 + 2 \times (-4) - 8 = 16 - 8 - 8 = 0$$

$$f(2) = (2)^2 + 2 \times 2 - 8 = 4 + 4 - 8 = 0$$

$$f(-4) = f(2)$$

Thus, there exists  $c \in (-4, 2)$  such that  $f'(c) = 0$

$$2c + 2 = 0$$

$$2c = -2$$

$$c = -1 \in (-4, 2)$$

Hence the theorem is verified.

# Mean Value Theorem

Let  $f: [a, b] \rightarrow \mathbb{R}$  be a continuous function on  $[a, b]$  and differentiable on  $(a, b)$ . Then there exists some 'c' in  $(a, b)$  such that  $f'(c) = \frac{f(b) - f(a)}{b - a}$

$$b - a$$

For example: Verify [Mean Value Theorem](#) for the function  $f(x) = x^2$  in the interval  $[2, 4]$

Solution: Since  $f(x) = x^2$ , it is continuous in  $[2, 4]$

$f'(x) = 2x$ , which exists in  $(2, 4)$

$$f(2) = 2^2 = 4 \text{ and } f(4) = 4^2 = 16$$

$$f'(c) = \frac{f(4) - f(2)}{4 - 2} = \frac{16 - 4}{2}$$

$$= \frac{12}{2} = 6$$

But according to MVT,  $f'(c) = 6$

$$2c = 6 \text{ [} f'(x) = 2x \text{]}$$

$$c = 3 \in (2, 4)$$

Hence the theorem is verified.

## Continuous functions

Suppose 'f' is a real function on a subset of the real numbers and let 'c' be a point in the domain of 'f'. Then 'f' is [continuous](#) at 'c' if

$$\lim_{x \rightarrow c} f(x) = f(c)$$

OR

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^+} f(x) = f(c)$$

All polynomial functions and trigonometric functions are continuous functions.

## Differentiable Functions

A function is said to be differentiable if its derivative exists. The derivative of a function is defined as

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

The derivative is also denoted by  $\frac{d}{dx} [f(x)]$ ,  $y'$ ,  $y_1$ ,  $\frac{dy}{dx}$

**Note:** Every differentiable function is continuous.

## Open and closed intervals

The [interval](#) which contains all the points between 'a' and 'b' excluding the extreme values is called open interval. It is denoted by (a, b) and is defined as  $(a, b) = \{x: a < x < b, x \in \mathbb{R}\}$ .

The interval which contains all the points between 'a' and 'b' including the extreme values is called closed interval. It is denoted by [a, b] and is defined as  $[a, b] = \{x: a \leq x \leq b, x \in \mathbb{R}\}$ .

Now try it yourself! Should you still need any help, [click here](#) to schedule live online session with e Tutor!

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### Reference Links:

- [http://en.wikipedia.org/wiki/Mean\\_value\\_theorem](http://en.wikipedia.org/wiki/Mean_value_theorem)
- [http://en.wikipedia.org/wiki/Rolle%27s\\_theorem](http://en.wikipedia.org/wiki/Rolle%27s_theorem)
- [http://en.wikipedia.org/wiki/Open\\_interval#Terminology](http://en.wikipedia.org/wiki/Open_interval#Terminology)
- [http://en.wikipedia.org/wiki/Continuous\\_function](http://en.wikipedia.org/wiki/Continuous_function)
- <http://www.analyzemath.com/polynomial2/polynomial2.htm>

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