Mean Value Theorem

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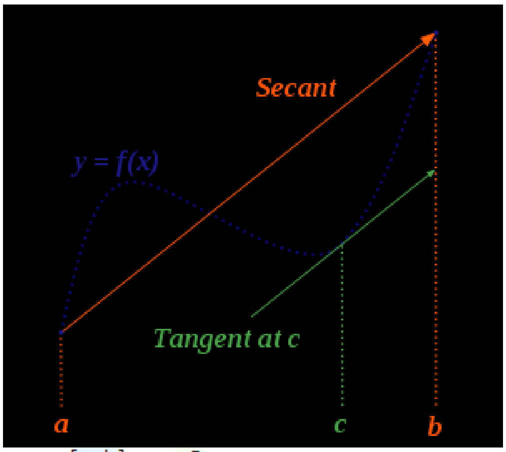
Rolle's Theorem



Let f: $[a, b] \longrightarrow R$ be continuous on [a, b] and differentiable in (a, b), such that f(a) = f(b), where 'a' and 'b' are some real numbers, then there exists some 'c' in (a, b) such that f'(c) = 0

For example: Verify Rolle's theorem for the function $f(x) = x^2 + 2x - 8$,   x ? [-4,2].
Solution: Since f(x) is a polynomial function, it is continuous in [-4,2],
f'(x) = 2x + 2 is differentiable in (-4, 2)
$f(-4) = (-4)^2 + 2 \times (-4) - 8 = 16 - 8 - 8 = 0$
$f(2) = (2)^2 + 2 \times 2 - 8 = 4 + 4 - 8 = 0$
f(-4) = f(2)
Thus, there exists c ? (-4, 2) such that f'(c) = 0
2c + 2 = 0
2c = -2
c = -1 ? (-4, 2)
Hence the theorem is verified.

Mean Value Theorem

Let f: $[a, b] \longrightarrow R$ be a continuous function on [a, b] and differentiable on (a, b). Then there exists some 'c' in (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

For example: Verify Mean Value Theorem for the function $f(x) = x^2$ in the interval [2, 4]
Solution: Since $f(x) = x^2$, it is continuous in [2, 4]
f'(x) = 2x, which exists in (2, 4)
$f(2) = 2^2 = 4$ and $f(4) = 4^2 = 16$

$$f'(c) = \frac{f(4) - f(2)}{4 - 2} = \frac{16 - 4}{2}$$

= 12/2 = 6
But according to MVT, f'(c) = 6
2c = 6 [f'(x) = 2x]
c = 3 ? (2, 4)

Hence the theorem is verified.

Continuous functions

Suppose 'f' is a real function on a subset of the real numbers and let 'c' be a point in the domain of 'f'. Then 'f' is continuous at 'c' if

$$\lim_{x \to c} f(x) = f(c)$$

OR

$$\lim_{x \to 1-} f(x) = \lim_{x \to 1+} f(x) = f(c)$$

All polynomial functions and trigonometric functions are continuous functions.

Differentiable Functions

A function is said to be differentiable if its derivative exists. The derivative of a function is defined as

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

The derivative is also denoted by $\frac{d}{dx}[f(x)]$, y', y_1, $\frac{dy}{dx}$

Note: Every differentiable function is continuous.

Open and closed intervals

The interval which contains all the points between 'a' and 'b' excluding the extreme values is called open interval. It is denoted by (a, b) and is defined as (a, b) = {x: a < x < b, x ? R}.
The interval which contains all the points between 'a' and 'b' including the extreme values is called closed interval. It is denoted by [a, b] and is defined as [a, b] = {x: a ? x ? b, x ? R}.

Now try it yourself! Should you still need any help, click here to schedule live online session with e Tutor!

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Reference Links:

- http://en.wikipedia.org/wiki/Mean_value_theorem
- http://en.wikipedia.org/wiki/Rolle%27s_theorem
- http://en.wikipedia.org/wiki/Open_interval#Terminology
- http://en.wikipedia.org/wiki/Continuous_function
- http://www.analyzemath.com/polynomial2/polynomial2.htm