RATIO TEST

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Introduction to the Ratio Test

$\sum_{n=0}^{\infty} a_n$ The comparison test and the limit comparison test hinge on first making a guess aboutconvergence and then finding an appropriate series for comparison, both of which can be difficult tasks in cases where informal principles cannot be applied. In such cases the next test can often be used, since it works exclusively with the terms of the given series- it requires neither an initial guess about convergence nor the discovery of a series for comparison.

Let ?u^k be a series with positive terms and suppose that $\rho = \lim_{k \to \infty} \frac{u_{k+1}}{u_k}$

a) If ? < 1, the series converges.

b) If ? > 1 or ?=+?, the series diverges.

c) If ? = 1, the series may converge or diverge, so that another test must be tried.

Problems Based On Ratio Test

Use the ratio test to determine whether the following series converge or diverge

1. ? (1/k!)

Solution:

$$\rho = \lim_{k \to +\infty} \frac{u_{k+1}}{u_k} = \lim_{k \to +\infty} \frac{[1/(k+1)!]}{1/k!}$$

$$= \lim_{k \to +\infty} \frac{k!}{(k+1)!}$$

$$= \lim_{k \to +\infty} \frac{1}{k+1} = 0 < 1$$

Since ? < 1, the series converges.

3. ? 1/[2k-1]

Solution: The ratio test is of no help since

$$\rho = \lim_{k \to +\infty} \frac{u_{k+1}}{u_k} = \lim_{k \to +\infty} \frac{1}{2(k+1)-1} \frac{2k-1}{1} = \lim_{k \to +\infty} \frac{2k-1}{2k+1} = 1$$

However, the integral test proves that the series diverges since

$${}_1\int^{+\infty} dx = \lim_{l \to +\infty} {}_1\int^{l} dx = \lim_{l \to +\infty} [\tfrac{1}{2} \ln(2x-1)]_1^{l} = +\infty$$

Both the comparison test and the limit comparison test would also have worked here.

Root Test

In cases where it is difficult or inconvenient to find the limit required for the ratio test, the next test is sometimes useful.
Let ?u_k be a series with positive terms and suppose that

$$\rho = \lim_{k \to +\infty} (u_k)^{1/k}$$

a) If ? < 1, the series converges.

b) If ? > 1, the series diverges

c) If ? = 1, the series may converge or diverge, so that another test must be tried.

Problems related to root test

Use the root test to determine whether the following series converge or diverge.

$$a)\ \Sigma \left(\frac{4k-5}{2k+1}\right)^k \qquad (b)\ \Sigma \frac{1}{[\ln(k+1)]^k}$$

Solution:

a) We have $\rho = \lim_{k \to +\infty} (u_k)^{1/k}$

$$= \lim_{k \to +\infty} \left(\frac{4k-5}{2k+1}\right)^k$$

$$= 2 > 1$$

Since ? > 1 we can say that the given series diverges.

b) We have $\rho = \lim_{k \to +\infty} (u^k)^{1/k}$

$$= \lim_{k \to +\infty} \frac{1}{\ln\{(k+1)^k\}^{1/k}}$$

$$= \lim_{k \to +\infty} \frac{1}{\ln(k+1)}$$

$$= 0 < 1$$

Since ? < 1, we can say that the series converges.

Now try it yourself! Should you still need any help, click here to schedule live online session with e Tutor!

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Reference Links:

- http://en.wikipedia.org/wiki/Ratio_test
- http://en.wikipedia.org/wiki/Root_test
- http://en.wikipedia.org/wiki/Convergent_series
- http://en.wikipedia.org/wiki/Divergent_series

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