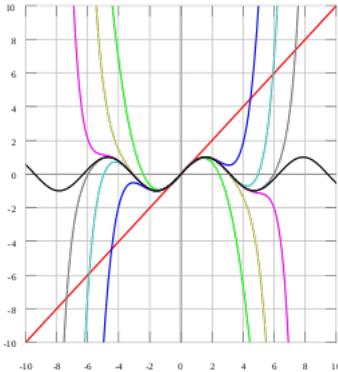


TAYLOR SERIES

Created: Saturday, 19 November 2011 06:33 | Published: Saturday, 19 November 2011 06:33 | Written by [Super User](#) | [Print](#)

Introduction



If 'f' has derivatives of all orders at x_0 , then we call the series

$$\sum \frac{f^{(k)}(x_0)}{k!} (x-x_0)^k = f(x_0) + f'(x_0)(x-x_0) + \frac{f''(x_0)}{2!}(x-x_0)^2 + \dots + \frac{f^{(k)}(x_0)}{k!}(x-x_0)^k + \dots$$

The Taylor series for 'f' about $x=x_0$.

In the special case where $x_0=0$, this series becomes

$$\sum \frac{f^{(k)}(0)}{k!} x^k = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \dots + \frac{f^{(k)}(0)}{k!}x^k + \dots$$

which is called as the Maclaurin series of 'f'.

Note that the nth Maclaurin and Taylor polynomials are the nth partial sums for the corresponding Maclaurin and Taylor series.

Example: Find the Maclaurin series for $1/(1-x)$

Solution: We know that the nth Maclaurin polynomial for $1/(1-x)$ is

$$p_n(x) = 1 + x + x^2 + \dots + x^n \quad (n=0, 1, 2, \dots)$$

Thus, the Maclaurin series for $1/(1-x)$ is

$$1 + x + x^2 + x^3 + \dots + x^k + \dots$$

Example: Find the Taylor series for $1/x$ about $x=1$

Solution: We know that the nth Taylor polynomial for $1/x$ about $x=1$ is

$$(-1)^k (x-1)^k = 1 - (x-1) + (x-1)^2 - (x-1)^3 + \dots + (-1)^n (x-1)^n$$

Thus the Taylor series for $1/x$ about $x=1$ is

$$(-1)^k (x-1)^k = 1 - (x-1) + (x-1)^2 - (x-1)^3 + \dots + (-1)^k (x-1)^k + \dots$$

Power Series in x

Maclaurin and Taylor series differ from the series that we have discussed above that their terms are not merely constants, but instead involve a variable. These are examples of power series which is defined as

If c_0, c_1, c_2, \dots Are constants and x is a variable then a series of the form

$$c_0 + c_1x + c_2x^2 + \dots + c_kx^k + \dots$$

is called a power series in x

Here are some examples

$$i) \sum x^k = 1 + x + x^2 + x^3 + \dots$$

$$ii) \sum \frac{x^k}{k!} = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots$$

$$iii) \sum \frac{(-1)^k x^{2k}}{(2k)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$$

Indeed every Maclaurin series is a power series in x.

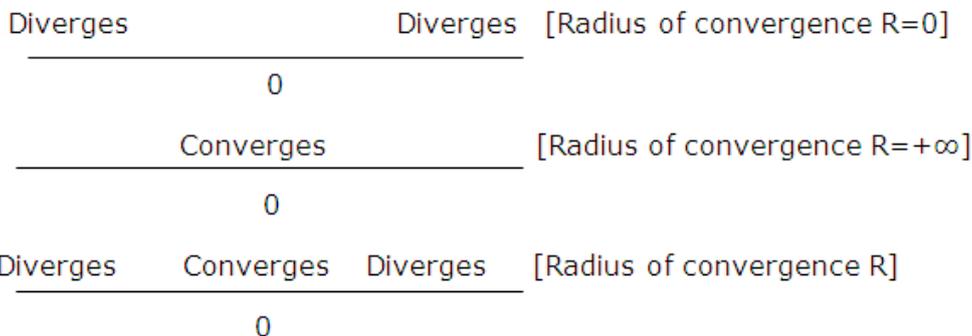
Radius and interval of convergence

If a numerical value is substituted for x in a power series $\sum c_k x^k$, then the resulting series of numbers may either converge or diverge. This leads to the problem of determining the set of x-values for which a given power series converges; this is called its convergence set.

For any a power series in x, exactly one of the following is true:

- The series converges only for x=0
- The series converges absolutely (and hence converges) for all real values of x
- The series converges absolutely (and hence converges) for all x in some finite open interval (-R, R) and diverge if x < -R or x > R. At either of the values x=R or x=-R, the series may converge absolutely, converge conditionally or diverge, depending on the particular series.

This theorem states that the convergence set for a power series in x is always as interval centered at x=0 (possibly just the value x=0 itself or possibly infinite). For this reason, the convergence set of a power series in x is called the interval of convergence. In the case where the convergence set is the single value x=0 we say that the series has radius of convergence 0, in the case where the convergence set is (-∞, ∞) we say that the series has radius of convergence ∞ and in the case where the convergence set extends between -R and R we say that the series has radius of convergence R.



The usual procedure for finding the interval of convergence of a power series is to apply the ratio test for absolute convergence.

The following example illustrates how this works.

Example: Find the interval of convergence and radius of convergence of the following power series $\sum x^k$ and $\sum x^k/k!$

Solution:

$\sum x^k$

We apply the ratio test for absolute convergence. We have

$$\rho = \lim_{k \rightarrow \infty} \left| \frac{u^{k+1}}{u^k} \right| = \lim_{k \rightarrow \infty} \left| \frac{x^{k+1}}{x^k} \right| = \lim_{k \rightarrow \infty} |x| = |x|$$

So the series converges absolutely if $\rho = |x| < 1$ and diverges if $\rho = |x| > 1$. The test is inconclusive if $|x|=1$ (x=1 or x=-1), which means that we will have to investigate convergence at these values separately. At these values the series becomes

$$\sum 1^k = 1 + 1 + 1 + 1 + 1 + \dots \quad [x=1]$$

$$\sum (-1)^k = 1 - 1 + 1 - 1 + 1 - 1 + \dots \quad [x=-1]$$

Both of which diverge, thus, the interval of convergence for the given power series is (-1, 1) and the radius of convergence is R=1.

$\sum x^k/k!$

Applying the ratio test for absolute convergence, we obtain

$$\rho = \lim_{k \rightarrow \infty} \left| \frac{u^{k+1}}{u^k} \right| = \lim_{k \rightarrow \infty} \left| \frac{x^{k+1} k!}{(k+1)! x^k} \right| = \lim_{k \rightarrow \infty} \left| \frac{x}{k+1} \right| = 0$$

Power Series in $x-x_0$

$\sum_{k=0}^{\infty} c_k(x-x_0)^k = c_0 + c_1(x-x_0) + c_2(x-x_0)^2 + \dots + c_k(x-x_0)^k + \dots$ is called a power series in $x-x_0$.

For a power series $\sum_{k=0}^{\infty} c_k(x-x_0)^k$, exactly one of the following statements is true:

- The series converges only for $x=x_0$.
- The series converges absolutely and hence converges for all real values of x .
- The series converges and hence converges for all x in some finite open interval (x_0-R, x_0+R) and diverges if $x < x_0-R$ or $x > x_0+R$. At either of the values $x=x_0-R$ or $x=x_0+R$, the series may converge absolutely, converge conditionally or diverge depending on the particular series.

It follows from the above statements that the set of values for which a power series in $x-x_0$ converges is always an interval centered at $x=x_0$; we call this the interval of convergence. In part (a) the interval of convergence reduces to the single value $x=x_0$ in which case we say that the series has radius of convergence $R=0$; in part (b) the interval of convergence is infinite, in which case we say that the series has radius of convergence $R=+\infty$; and in part (c) the interval extends between x_0-R and x_0+R , in which case we say that the series has radius of convergence R .

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Reference Links:

- http://en.wikipedia.org/wiki/Taylor_series
- <http://www.intmath.com/series-expansion/2-maclaurin-series.php>
- http://en.wikipedia.org/wiki/Power_series
- http://en.wikipedia.org/wiki/Radius_of_convergence

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