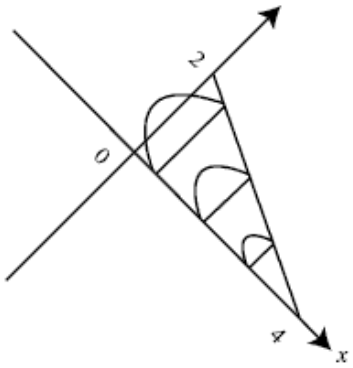
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VOLUME OF A SOLID WITH KNOWN CROSS SECTION

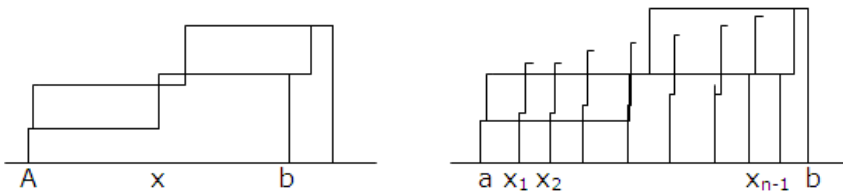
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Introduction



We have already learned to find the area of a plane region bounded by two curves which is obtained by integrating the length of a general cross section over an appropriate interval. Here we will see that the same basic principle can be used to find volumes of certain three dimensional solids.

Let S be a solid that extends along the x-axis and is bounded on the left and right, respectively, by the planes that are perpendicular to the x-axis at x=a and x=b. We are finding the volume V of the solid, assuming that its cross-sectional area A(x) is known at each x in the interval [a, b].
To solve this problem we divide the interval [a, b] into n subintervals, which has the effect of dividing the solid into n slabs [fig (ii)]



If we assume that the width of the k^{th} slab is $?x_k$, then the volume of the slab can be approximated by the volume of a right cylinder of width (height) $?x_k$ and cross-sectional area $A(x_k^*)$, where x_k^* is a number in the k^{th} subinterval. Adding these approximations yields the following Riemann sum that approximates the volume V:

$$V \approx \sum A(x_k^*)?x_k$$

Taking the limit as n increases and the widths of the subintervals approach zero yields the definite integral

I. $$V = \lim_{\max ?x_k \to 0} \sum ?A(x_k^*)?x_k = \int_a^b A(x)dx$$

We can conclude the result in the following way,

Volume formula

Let S be a solid bounded by two parallel planes perpendicular to the x-axis at x=a and x=b. If, for each x in [a, b] the cross-sectional area of S perpendicular to the x-axis is A(x), then the volume of the solid is,

$$V = \int_a^b A(x)\,dx$$ provided A(x) is integrable.

Volume Formula

Let S be a solid bounded by two parallel planes perpendicular to the y-axis at y=c and y=d. If, for each y in [c, d], the cross-sectional area of S perpendicular to the y-axis is A(y), then the volume of the solid is,

$$V = \int_c^d A(y)\,dy$$ provided A(y) is integrable.

In words, these formulas states that, “The volume of a solid can be obtained by integrating the cross-sectional area from

one end of the solid to the other".

Example: Find the formula for the volume of a right pyramid whose altitude is h and whose base is a square with sides of length a.

Solution: We introduce a rectangular coordinate system in which the y-axis passes through apex and is perpendicular to the base, and the x-axis passes through the base and is parallel to a side of the base.

At any 'y' in the interval [0, h] on the y-axis, the cross section perpendicular to the y-axis is a square. If 's' denotes the length of a side of this square, then by similar triangles.

$$\frac{\frac{1}{2}s}{\frac{1}{2}a} = \frac{h-y}{h}$$

$$s = (a/h)(h-y)$$

Thus the area A(y) of the cross section at y is

$$A(y) = s^2 = (a^2/h^2)(h-y)^2$$

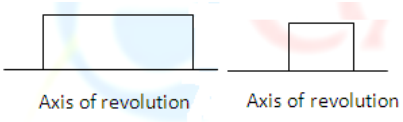
$$V = \int_0^h A(y)dy = \int_0^h (a^2/h^2)(h-y)^2 = (a^2/h^2)\int_0^h (h-y)^2 dy$$

$$= (a^2/h^2)[-1/3(h-y)^3]_0^h = 1/3a^2h$$

Thus the volume is 1/3 of the area of the base times the altitude.

Solids of revolution

A[solid of revolution] is a solid that is generated by revolving a plane region about a line that lies in the same plane as the region; the line is called the axis of revolution.



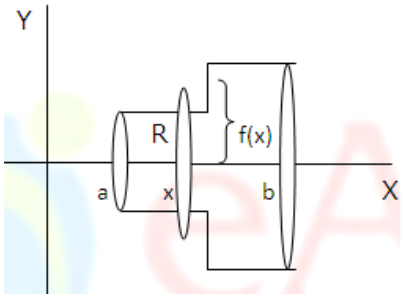
Volume of a solid of revolution

Let f be continuous and non-negative on [a, b] and let R be the region that is bounded by y=f(x), below by the x-axis, and on the sides by the lines x=a and x=b, then the volume of the solid or revolution that is generated by revolving the region R about the x-axis is given by

$$V = \int_a^b [f(x)]^2 dx$$

$$= \int_a^b y^2 dx$$

$$= \int_a^b y^2 dx$$



Example: Find the volume of a paraboloid of revolution formed by revolving the parabola $y^2=8x$ about the x-axis from x=0 to x=6
Solution: The equation of the parabola is $y^2=8x$,
Hence volume = $_0?^6$ $?y^2dx$
= $?_0?^6$ 8x dx
= $8?[x^2/2]_0^6$
= 8? x ½ $[6^2-0^2]$
=4?x36
= 144?cubic units.

Volume by cylindrical shells

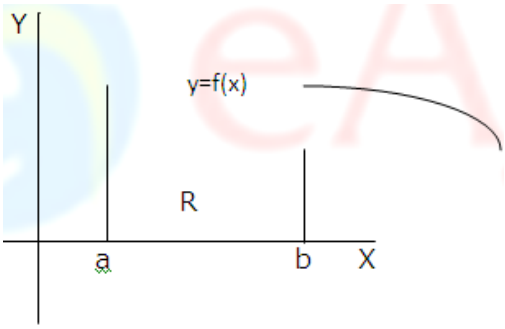
A cylindrical shell is a solid enclosed by two concentric right circular cylinders. The volume V of a cylindrical shell with inner radius r_1, outer radius r_2, and height h can be written as

V = (area of cross section).height

Let f be continuous and non-negative on [a, b] and let R be the region that is bounded above by y=f(x) below by the x-axis, and on the sides by the lines x=a and x=b. Then the volume V of the solid revolution that is generated by revolving the region R about the y-axis is given by

V = $_a?^b$ 2?xf(x)dx

A.



Example: Use cylindrical shells to find the volume of the solid generated when the region R in the first quadrant enclosed between y=x and $y=x^2$ is revolved about the y-axis.
Solution: V= $_0?^1$ 2?x(x-x^2)dx = 2? $_0?^1$ (x^2-x^3)dx
= 2? [(1/3) – (1/4)]
= ?/6 cubic units.

Now try it yourself! Should you still need any help, click here to schedule live online session with e Tutor!

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