ALGEBRA OF COMPLEX NUMBERS

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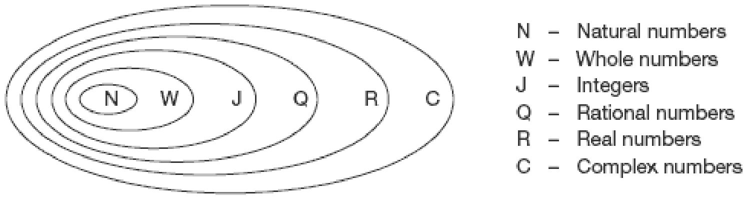
Introduction



A number of the form z=a + ib, i=?-1, where ‘a’ and ‘b’ are real numbers is called acomplex number. Here ‘a’ is the real part and ‘b’ is the imaginary part of ‘z’.

- The real part is denoted by Re z and imaginary part by Im z.
- If the real part is zero then is is said to be purely imaginary and if the imaginary part is zero then it is said to be purely real.
- All real numbers are complex numbers with imaginary part zero.

For example: 5= 5 + i0

-3 =-3 + i0

Two complex numbers z_1=a + ib and z_2=c + id are equal if a=c and b=d

If two complex numbers are equal then their real parts are equal and imaginary parts are also equal.

Example: If 4x+ i(4y-x) = 8+ i6, then find ‘x’ and ‘y’

Solution: Since the complex numbers are equal, we have

4x=8 and 4y-x=6

4x=8

x=2

4y-x =6 becomes 4y-2 =6

4y=6+2=8

y=2

Hence x=2 and y=2

Algebra of Complex Numbers

In algebra of complex numbers we deal with addition of two complex numbers, subtraction of two complex numbers, multiplication of two complex numbers , division of two complex numbers and the properties satisfied by them.

Addition of two complex numbers

Let z_1= a + ib and z_2= c + id be any two complex numbers, then the sum z_1+z_2 = (a+c) + i(b+d), which is again a complex number. While adding two complex numbers, we have to add real parts together ad imaginary parts together.
Properties satisfied by addition of complex numbers.

- Closure Property: The sum of two complex numbers is a complex number.
- Commutative Property: For any two complex numbers z_1 and z_2, $z_1+z_2= z_2+z_1$

- Associative Property: For any three complex numbers z_1, z_2 and z_3, $z_1+(z_2+z_3) = (z_1+z_2)+z_3$
- Existence of additive identity: For every complex number 'z' there exists a complex number 0+i0 (denoted as 0) such that z+0=z, which is called the additive identity.
- Existence of additive inverse: To every complex number z=a +ib, we can find a complex number –z = (-a)+(-ib) [denoted as –z] called the additive inverse such that z + (-z) =0.

Difference of two complex numbers

Given two complex numbers z_1 and z_2, the difference $z_1 - z_2$ is defined as follows, $z_1 - z_2 = z_1 + (-z_2)$

All the above mentioned properties are satisfied here also.

Example: Evaluate (1-i)-(-1+i6)

Solution: (1-i) – (-1+i6) = (1-i) + (1-i6)

= [1+1] + [-i+-i6]

= 2 + (-i7)

= 2 + -7i

= 2 – 7i

Multiplication of two complex numbers

Let z_1=a + ib and z_2=c + id be any two complex numbers. Then, the product z_1z_2 is defined as follows,

z_1z_2 = (ac-bd) + i(ad+bc)

Multiplication of complex numbers satisfies the following properties.

- Closure Property: The product of two complex numbers is again a complex number.
- Commutative Property: For any two complex numbers z_1 and z_2, $z_1z_2= z_2z_1$
- Associative Property: For any three complex numbers z_1, z_2 and z_3, $(z_1z_2)z_3 = z_1(z_2z_3)$
- Existence of multiplicative identity: To every complex number z, there exists 1_i0 (denoted as 1), called the multiplicative identity such that z.1 = 1.z =z.
- Existence of multiplicative inverse: For every non-zero complex number z we have 1/z or z-1, called the multiplicative inverse of z such that z.1/z=1
- Distributive Law: For any three complex numbers z_1, z_2, z_3

a) $z_1(z_2+z_3) = z_1z_2 + z_1z_3$

b) $(z_1+z_2)z_3 = z_1z_3 + z_2z_3$

Division of two complex numbers

Given any two complex numbers z_1 and z_2, where z_2?0, the quotient z_1/z_2 is defined by z_1. $1/z_2$

Here we multiply by the conjugate of the denominator to get the answer.

Example: Find the value of $\frac{2+3i}{1+i}$

Solution: $\frac{2+3i}{1+i} = \frac{(2+3i)\ (1-i)}{(1+i)\ (1-i)}$

$= \frac{2-2i+3i-3i^2}{1-i+i-i^2}$

$= \frac{2+i+3}{1+1}$

$= \frac{5+i}{2}$

= 5/2 + ½ i

Example: Find the multiplicative inverse of ?5+3i

Solution: The multiplicative inverse of ?5 + 3i is $\frac{1}{\sqrt{5} + 3i}$

$$\frac{1}{\sqrt{5} + 3i} = \frac{\sqrt{5} - 3i}{(\sqrt{5}-3i)(\sqrt{5}+3i)}$$

$$= \frac{\sqrt{5} - 3i}{5 - 9i^2}$$

$$= (\sqrt{5} - 3i)/14$$

$$= \frac{\sqrt{5}}{14} - i\,\frac{3}{14}$$

Hence the multiplicative inverse of ?5 + 3i is (?5/14) + i(3/14)

Now try it yourself! Should you still need any help, click here to schedule live online session with e Tutor!

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Reference Links:

- http://en.wikipedia.org/wiki/Complex_number
- http://en.wikipedia.org/wiki/Linear_equation
- http://en.citizendium.org/wiki/Complex_number
- http://en.wikipedia.org/wiki/Multiplicative_inverse
- http://en.wikipedia.org/wiki/Associativity

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