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Argand Plane and Polar Representation of a Complex Number

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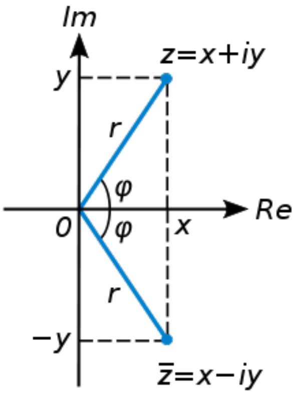
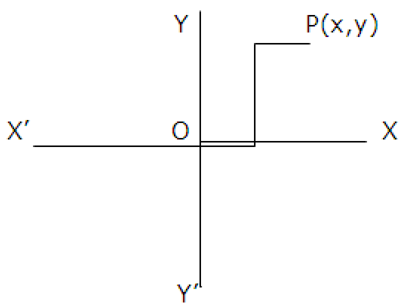
Argand Plane



We have already learned that corresponding to each ordered pair of real numbers (x, y) we get a unique point in XY plane with reference to a set of mutually perpendicular lines known as X-axis and Y-axis. The complex number z=x + iy which corresponds to the ordered pair (x, y) can be represented geometrically as the unique point P(x, y) in the XY plane.
The plane having a complex number assigned to each of its point is called the Complex Plane or Argand Plane.
The point on x-axis corresponds to the complex numbers of the form a+i0 and the point on the y-axis corresponds to the complex numbers of the form 0 + ib.
The x-axis is called as real axis and y-axis is called as imaginary axis in the Argand Plane.
The representation of a complex number z=x + iy and its conjugate z= x- iy in the Argand Plane are respectively the points (x, y) and (x, -y). Clearly (x, -y) is the mirror image of the point (x, y) on the real axis.

In the Argand Plane, the modulus of a complex number x + iy = ?x^2 + y^2 is the distance between the point P(x, y) and the origin O(0, 0).



Here OP = ?x^2+y^2

Polar Representation of a Complex Number

Let the point P represent the non-zero complex number z = x +iy. Let the modulus of z be r and ? be the angle made by OP with positive direction of x-axis. Then we can note that the point P is determined by the ordered pair (r, ?), called the polar co-ordinates. Here we consider the origin as the pole and the positive x-axis as the initial line.
So, we have, x = rcos? and y = rsin? and hence z = x + iy becomes z=r(cos? +isin?), which is called polar representation of the complex number z

Here r=|z|=?x^2+y^2 and the value of ? lies between 0 and 2? which is called as argument or amplitude of z and is denoted by argz. In other words, if z=x + iy is a complex number then its polar form is given by z=r(cos? + isin?), where ‘r’ is the modulus and ? is the amplitude or argument.

Easy method for finding the argument, ?

We can easily find the modulus ‘r’ but the value of ‘?’ varies according to the different quadrants. While finding the value of ‘?’, we must be aware of the standard angles and its values otherwise we find it difficult.
Let ? be the standard angle, if it comes in first quadrant then we can directly substitute its value. In other cases, we have to transform it to the corresponding standard value which we have learned in class XI. To overcome this difficulty here are some formulas to find the argument ‘?’
If ‘?’ lies in first quadrant take ? = ?
Second quadrant, ? = ?-?
Third quadrant, ? = - (?-?)
Fourth quadrant, ? = -?
Below given are few examples
1. Convert ?3+i in the polar form
Let z= x + iy = ?3 +i
We know x=?3 and y=1
Again we have r=?(?3)2 + 12
= 2
rcos? = ?3 and rsin? = 1
2cos?=?3 and 2sin? = 1
Cos? = ?3/2 and sin? = ½
Both cos? and sin? are positive, so ‘?’ belongs to 1st quadrant.
The value of ‘?’ for which cos?=?3/2 and sin?=1/2 is 30? or ?/6
Hence r=2 and ?=?/6
Polar form of ?3+i = 2[cos?/6 + isin?/6]
2. Find the modulus and argument of the complex number -1-i
Let z= x +iy = -1-i
x=-1 and y=-1
r= ?x^2+y^2 = ?(-1)2+(-1)2
= ?1+1 =?2
Modulus, r= ?2
Here ?2 cos?= -1 and ?2 sin? = -1
Cos? =-1/?2 and sin?=-1/?2
Both cos? and sin? are negative, so ? belongs to third quadrant.
The value of ? for which cos? = sin? =1/?2 is ?/4 [without sign]
Hence the argument = - (?-?) = - (?-?/4) = $\frac{3\pi}{4}$

Argument = 3?/4

Fundamental theorem of Algebra

Fundamental theorem of Algebra states that “A polynomial equation of degree n has n roots”
The above statement tells us that a linear equation has one root, quadratic equation has two roots, cubic equation has three roots, fourth degree equation has four roots and so on.

Quadratic Equations

We know that the general quadratic equation is of the form ax2+bx+c=0, a?0. This equation has two roots and the nature of roots has been studied in earlier classes. Here we consider the case where the discriminant, $b^2-4ac<0$.

We have learned that square root of negative real numbers is set of complex numbers, therefore, the solution of $ax^2+bx+c=0$ is given by

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{-b \pm \sqrt{4ac-b^2}\, i}{2a}$$

Example: Solve $\sqrt{2}x^2+x+\sqrt{2}=0$

Here $a=\sqrt{2}$, $b=1$ and $c=\sqrt{2}$

$$b^2 - 4ac = 1^2-4\times\sqrt{2}\times\sqrt{2}$$

$$= 1 - 8$$

$$= -7 < 0$$

$$x = \frac{-1 \pm \sqrt{(-7)}}{2\sqrt{2}}$$

$$= \frac{-1 \pm \sqrt{7}\, i}{2\sqrt{2}}$$

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Reference links:

- http://en.wikipedia.org/wiki/Ordered_pair
- http://en.wikipedia.org/wiki/Complex_plane
- http://en.wikipedia.org/wiki/Quadratic_equation
- http://en.citizendium.org/wiki/Fundamental_Theorem_of_Algebra
- http://www.intmath.com/complex-numbers/4-polar-form.php

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