Modulus and the Conjugate of a Complex Number

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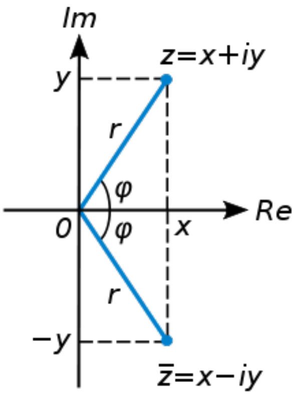
Introduction



The modulus of a complex number denotes its magnitude and conjugate of a complex number is obtained by changing the sign of imaginary part of a complex number. If the imaginary part is positive we make it negative and if it is negative, we make it positive. The sign of real part doesn't change.

Let z=a + ib be a complex number, then the modulus of 'z' is denoted by |z| and is defined as |z| = ?a^2+b^2

For example: 1) |5-i| = ?5^2 + (-1)^2

= ?25 + 1

= ?26

2) |-3+4i| = ?(-3)2 + 42

= ?9+16

= ?25

= 5

The conjugate of a complex number z=a + ib is denoted by z and is defined as z = a-ib

Clearly we can observe that $z\bar{z}$ = |z|2

Let z = a + ib then $\bar{z}$= a − ib

$$z\bar{z} = (a + ib)(a - ib)$$

$$= a^2 - i^2b^2$$

$$= a^2 + b^2 \qquad [i^2=-1]$$

$$= [\sqrt{a^2+b^2}]^2$$

$$= |z|^2$$

Properties of Modulus and conjugate of complex numbers

For any two complex numbers z1 and z2, we have

i) $|z_1 z_2| = |z_1|\ |z_2|$

ii) $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$ provided $|z_1| \neq 0$

iii) $\overline{z_1 z_2} = \overline{z_1}\ \overline{z_2}$

iv) $\overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}$

Square Root of a negative real number

We know i^2=-1 and $(–i)^2=i^2$ =-1
Hence the square roots of -1 are i, -i but the symbol ?-1 mean 'i' only.
Now we can see that i and –i both are the solutions of the equation x^2+1=0
Similarly, $(?5i)^2 = 5i^2$ = 5 x -1 = -5
$(-?5i)^2 = 5i^2$ = 5 x -1 = -5
Generally, if 'a' is a positive real number, ?-a = ?a ?-1 = ?a i
We have already learned that ?a x ?b = ?ab for all positive real numbers 'a' and 'b'
We can examine this result for i2,
i^2 = i x i = ?-1 x ?-1 = ?-1 x -1 = ?1 =1, which is a contradiction to the fact that i^2=-1
Hence ?a x ?b ? ?ab if both 'a' and 'b' are negative real numbers
If 'a' and 'b' are zero, then ?a x ?b = ?ab =0

Powers of 'i'

We know that i^2 = -1, different powers of 'i' are given below
$i^3 = i^2$ x i = -1 x i = -i
$i^4 = (i^2)2 = (-1)^2$ = 1
$i^5 = i^4$ x i = 1 x i = i
$i^6 = i^5$ x i = i x i = i^2 = -1 and so on.

$$i^{-1} = \frac{1}{i} = \frac{1}{i}\left(\frac{i}{i}\right) = \frac{i}{-1} = -i$$

$$i^{-2} = \frac{1}{i^2} = \frac{1}{-1} = -1$$

$$i^{-3} = \frac{1}{i^3} = \frac{1}{-i} \times \frac{i}{i} = \frac{i}{1} = i$$

Also we have,

$$i^{-4} = \frac{1}{i^4} = \frac{1}{1} = 1$$

In general, for any integer k, i4k=1, i4k+1=i, i4k+2 = -1, i4k+3 = -i

Identities in Complex numbers

For any two complex numbers z1 and z2 we have the followingidentities

- $(z_1 + z_2)^2 = z_1^2 + z_2^2 + 2z_1z_2$

$(z_1 + z_2)^2 = (z_1 + z_2)(z_1+z_2)$
$= z_1^2 + z_1z_2 + z_1z_2 + z_2^2$
$= z_1^2 + 2z_1z_2 + z_2^2$

- $(z_1-z_2)2 = z_1^2 -2z_1z_2+z_2^2$
- (z1+z2)3 = z13+3z12z2+3z1z22+z23
- (z1-z2)3 = z13-3z12z2+3z1z22+z23
- z12-z22= (z1+z2)(z1-z2)

Express in the form a + ib
1) $i^9 + i^{19}$

1) $i^9 + i^{19} = i^8\, i + i^{18}\, i$

$= 1 \times i + -1 \times i$

$= i + -i$

$= 0$

$= 0 + i0$

Now try it yourself! Should you still need any help, click here to schedule live online session with e Tutor!

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Reference Links:

- http://en.wikipedia.org/wiki/Modulus_of_complex_number
- http://tvwiki.tv/wiki/Square_root
- http://www.cut-the-knot.org/arithmetic/algebra/ComplexNumberIdentities.shtml
- http://www.purplemath.com/modules/complex.htm
- http://en.wikipedia.org/wiki/Complex_conjugate

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