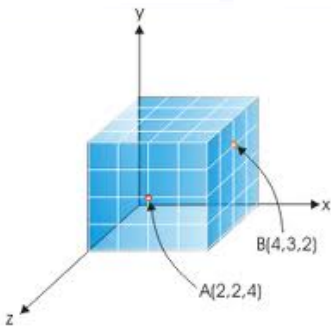


Reducing Cartesian Form of a line to Vector Form and vice-versa

Created: Wednesday, 23 November 2011 07:48 | Published: Wednesday, 23 November 2011 07:48 | Written by [Super User](#) | [Print](#)

Cartesian Form of a line passing through a given point



Equation of a line passing through a given point $P(x_1, y_1, z_1)$ and parallel to a given vector b having direction ratios $\langle a, b, c \rangle$ is

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$$

If $\langle l, m, n \rangle$ are the [direction cosines](#) then its equation is given by

$$\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$$

Reduction of Cartesian form to the Vector form

If $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$ is the equation of line then equate this to a

constant, say λ to get the vector form.

$$\text{So, } \frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c} = \lambda$$

$$x = a\lambda + x_1, y = b\lambda + y_1 \text{ and } z = c\lambda + z_1$$

$$\text{But } \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\vec{r} = (a\lambda + x_1)\hat{i} + (b\lambda + y_1)\hat{j} + (c\lambda + z_1)\hat{k}$$

$$= (x_1\hat{i} + y_1\hat{j} + z_1\hat{k}) + \lambda(a\hat{i} + b\hat{j} + c\hat{k})$$

$$= \vec{a} + \lambda\vec{b}, \text{ where } \vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k} \text{ and } \vec{b} = a\hat{i} + b\hat{j} + c\hat{k}$$

Hence the vector equation of line passing through a point with position vector \vec{a} and parallel to \vec{b} is $\vec{r} = \vec{a} + \lambda\vec{b}$

Reduction of Vector form to the Cartesian Form

If $\vec{r} = \vec{a} + \lambda \vec{b}$ is the Vector form of a line then its Cartesian equation is obtained by substituting $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ and $\vec{b} = a\hat{i} + b\hat{j} + c\hat{k}$

$$\vec{r} = \vec{a} + \lambda \vec{b}$$

$$x\hat{i} + y\hat{j} + z\hat{k} = (x_1\hat{i} + y_1\hat{j} + z_1\hat{k}) + \lambda(a\hat{i} + b\hat{j} + c\hat{k})$$

$$x\hat{i} + y\hat{j} + z\hat{k} = (x_1 + \lambda a)\hat{i} + (y_1 + \lambda b)\hat{j} + (z_1 + \lambda c)\hat{k}$$

$$x = x_1 + \lambda a, \quad y = y_1 + \lambda b, \quad z = z_1 + \lambda c$$

$$x - x_1 = \lambda a, \quad y - y_1 = \lambda b, \quad z - z_1 = \lambda c$$

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c} = \lambda, \text{ which is the Cartesian form}$$

Cartesian equation of a line passing through two points

If P(x₁, y₁, z₁) and Q(x₂, y₂, z₂) are two given points then Cartesian equation of line is

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

Reduction of Cartesian form to Vector form

As in the previous case, equate the Cartesian form of the line to λ , so that it

$$a. \quad \frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1} = \lambda$$

$$x - x_1 = \lambda(x_2 - x_1), \quad y - y_1 = \lambda(y_2 - y_1), \quad z - z_1 = \lambda(z_2 - z_1)$$

$$x = x_1 + \lambda(x_2 - x_1), \quad y = y_1 + \lambda(y_2 - y_1), \quad z = z_1 + \lambda(z_2 - z_1)$$

$$x\hat{i} + y\hat{j} + z\hat{k} = [x_1 + \lambda(x_2 - x_1)]\hat{i} + [y_1 + \lambda(y_2 - y_1)]\hat{j} + [z_1 + \lambda(z_2 - z_1)]\hat{k}$$

$$x\hat{i} + y\hat{j} + z\hat{k} = [x_1\hat{i} + y_1\hat{j} + z_1\hat{k}] + \lambda[(x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}]$$

becomes $x\hat{i} + y\hat{j} + z\hat{k} = [x_1\hat{i} + y_1\hat{j} + z_1\hat{k}] + \lambda[(x_2\hat{i} + y_2\hat{j} + z_2\hat{k}) - (x_1\hat{i} + y_1\hat{j} + z_1\hat{k})]$

$$\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a}), \text{ which is the vector equation where } \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}, \\ \vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k} \text{ and } \vec{b} = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$$

Reduction of Vector form to Cartesian form

If $\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$ is the vector equation of a line passing through the points with position vectors \vec{a} and \vec{b} then take $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ and $\vec{b} = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$ to obtain its Cartesian form.

$$\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a}) \text{ becomes}$$

$$x\hat{i} + y\hat{j} + z\hat{k} = (x_1\hat{i} + y_1\hat{j} + z_1\hat{k}) + \lambda[(x_2\hat{i} + y_2\hat{j} + z_2\hat{k}) - (x_1\hat{i} + y_1\hat{j} + z_1\hat{k})]$$

$$(x - x_1)\hat{i} + (y - y_1)\hat{j} + (z - z_1)\hat{k} = \lambda[(x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}]$$

$$x - x_1 = \lambda(x_2 - x_1), \quad y - y_1 = \lambda(y_2 - y_1) \text{ and } z - z_1 = \lambda(z_2 - z_1) \text{ [Equating like terms]}$$

$$\frac{x - x_1}{x_2 - x_1} = \lambda, \quad \frac{y - y_1}{y_2 - y_1} = \lambda, \quad \text{and} \quad \frac{z - z_1}{z_2 - z_1} = \lambda$$

Hence from the above three equations, we can write

b.

$$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1} = \lambda, \text{ which is the Cartesian form.}$$

Angle between two lines

If $\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu \vec{b}_2$ are the vector equations of two lines then [angle](#) between them is given by

$$\cos\theta = \frac{|\vec{b}_1 \cdot \vec{b}_2|}{|\vec{b}_1| |\vec{b}_2|}$$

If $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ and $\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$ are the Cartesian

Equations of two lines then angle between them is given by

$$\cos\theta = \frac{|a_1 a_2 + b_1 b_2 + c_1 c_2|}{\sqrt{a_1^2 + b_1^2 + c_1^2} \sqrt{a_2^2 + b_2^2 + c_2^2}}$$

Condition for parallelism and perpendicularity

- If the lines are [parallel](#) then $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$
- If the lines are [perpendicular](#) then $a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$

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Reference Links:

- <http://en.wikipedia.org/wiki/Angle>
- http://en.wikipedia.org/wiki/Direction_cosine
- [http://en.wikipedia.org/wiki/Parallel_\(geometry\)](http://en.wikipedia.org/wiki/Parallel_(geometry))
- <http://en.wikipedia.org/wiki/Perpendicular>

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