Reducing Cartesian Form of a line to Vector Form and vice-versa

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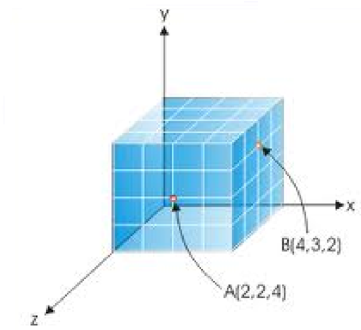
Cartesian Form of a line passing through a given point



Equation of a line passing through a given point P(x1, y1, z1) and parallel to a given vector b having direction ratios <a, b, c> is

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$$

If <l, m, n> are the direction cosines then its equation is given by

$$\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$$

Reduction of Cartesian form to the Vector form

If $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$ is the equation of line then equate this to a

constant, say ? to get the vector form.

So, $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c} = ?$

$x=a\lambda+x_1,\ y=b\lambda+y_1$ and $z=c\lambda+z_1$

But $\bar{r}=x\hat{i}+y\hat{j}+zk$

$\bar{r}= (a\lambda+x_1)\hat{i}+(b\lambda+y_1)\hat{j}+(c\lambda+z_1)k$

$= (x_1\hat{i}+y_1\hat{j}+z_1k) +\lambda(a\hat{i}+b\hat{j}+ck)$

$= \bar{a}+\lambda\bar{b}$, where $\bar{a}=x_1\hat{i}+y_1\hat{j}+z_1k$ and $\bar{b}=a\hat{i}+b\hat{j}+ck$

Hence the vector equation of line passing through a point with position vector a and parallel to b is r=a+?b

Reduction of Vector form to the Cartesian Form

If $\bar{r}=\bar{a}+\lambda\bar{b}$ is the Vector form a line then its Cartesian equation is obtained by substituting $\bar{r}= x\hat{i}+y\hat{j}+zk$, $\bar{a}=x_1\hat{i}+y_1\hat{j}+z_1k$ and $\bar{b}=a\hat{i}+b\hat{j}+ck$

$$\bar{r}=\bar{a}+\lambda\bar{b}$$

$$x\hat{i}+y\hat{j}+zk=(x_1\hat{i}+y_1\hat{j}+z_1k)+\lambda(a\hat{i}+b\hat{j}+ck)$$

$$x\hat{i}+y\hat{j}+zk=(x_1+\lambda a)\hat{i}+(y_1+\lambda b)\hat{j}+(z_1+\lambda c)k$$

$$x=x_1+\lambda a,\ y=y_1+\lambda b,\ z=z_1+\lambda c$$

$$x-x_1=\lambda a,\ y-y_1=\lambda b,\ z-z_1=\lambda c$$

$$\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}=$$?, which is the Cartesian form

Cartesian equation of a line passing through two points

If P(x1, y1, z1) and Q(x2, y2, z2) are two given points then Cartesian equation of line is

$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}$$

Reduction of Cartesian form to Vector form

As in the previous case, equate the Cartesian form of the line to ?, so that it

a.

$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}=\lambda$$

$$x-x_1=\lambda(x_2-x_1),\ y-y_1=\lambda(y_2-y_1),\ z-z_1=\lambda(z_2-z_1)$$

$$x=x_1+\lambda(x_2-x_1),\ y=y_1+\lambda(y_2-y_1),\ z=z_1+\lambda(z_2-z_1)$$

$$x\hat{i}+y\hat{j}+zk=[x_1+\lambda(x_2-x_1)]\hat{i}+[y_1+\lambda(y_2-y_1)]\hat{j}+[z_1+\lambda(z_2-z_1)]k$$

$$x\hat{i}+y\hat{j}+zk=[x_1\hat{i}+y_1\hat{j}+z_1k]+\lambda[(x_2-x_1)\hat{i}+(y_2-y_1)\hat{j}+(z_2-z_1)k]$$

becomes

$$x\hat{i}+y\hat{j}+zk=[x_1\hat{i}+y_1\hat{j}+z_1k]+\lambda[(x_2\hat{i}+y_2\hat{j}+z_2k)-(x_1\hat{i}+y_1\hat{j}+z_1k)]$$

$\bar{r}=\bar{a}+\lambda(\bar{b}-\bar{a})$, which is the vector equation where $\bar{r}=x\hat{i}+y\hat{j}+zk$, $\bar{a}=x_1\hat{i}+y_1\hat{j}+z_1k$ and $\bar{b}=x_2\hat{i}+y_2\hat{j}+z_2k$

Reduction of Vector form to Cartesian form

If $\bar{r}=\bar{a}+\lambda(\bar{b}-\bar{a})$ is the vector equation of a line passing through the points with position vectors $\bar{a}$ and $\bar{b}$ then take $\bar{r}=x\hat{i}+y\hat{j}+zk$, $\bar{a}=x_1\hat{i}+y_1\hat{j}+z_1k$ and $\bar{b}=x_2\hat{i}+y_2\hat{j}+z_2k$ to obtain its Cartesian form.

$\bar{r}=\bar{a}+\lambda(\bar{b}-\bar{a})$ becomes

$$x\hat{i}+y\hat{j}+zk=(x_1\hat{i}+y_1\hat{j}+z_1k)+\lambda[(x_2\hat{i}+y_2\hat{j}+z_2k)-(x_1\hat{i}+y_1\hat{j}+z_1k)]$$

$$(x-x_1)\hat{i}+(y-y_1)\hat{j}+(z-z_1)k=\lambda[(x_2-x_1)\hat{i}+(y_2-y_1)\hat{j}+(z_2-z_1)k]$$

$x-x_1=\lambda(x_2-x_1)$, $y-y_1=\lambda(y_2-y_1)$ and $z-z_1=\lambda(z_2-z_1)$ [Equating like terms]

$x-x_1 = \lambda$, $y-y_1 = \lambda$, and $z-z_1 = \lambda$

Hence from the above three equations, we can write

b.
$$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1} =$$?, which is the Cartesian form.

Angle between two lines

If $\bar{r}=\bar{a}_1+\lambda\bar{b}_1$ and $\bar{r}=\bar{a}_2+\mu\bar{b}_2$ are the vector equations of two lines then angle between them is given by

$$\cos\theta = \left|\frac{\bar{b}_1.\bar{b}_2}{|\bar{b}_1||\bar{b}_2|}\right|$$

If $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ and $\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$ are the Cartesian

Equations of two lines then angle between them is given by

$$\mathrm{Cos}\theta = \left|\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\,\sqrt{a_2^2+b_2^2+c_2^2}}\right|$$

Condition for parallelism and perpendicularity

- If the lines are parallel then $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$
- If the lines areperpendicular then a1a2+b1b2+c1c2=0

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Reference Links:

- http://en.wikipedia.org/wiki/Angle
- http://en.wikipedia.org/wiki/Direction_cosine
- http://en.wikipedia.org/wiki/Parallel_(geometry)
- http://en.wikipedia.org/wiki/Perpendicular