## ADJOINT AND INVERSE OF A MATRIX

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## Co-factors

$A^{-1}=\frac{\operatorname{adj}(A)}{|A|}$ It is a square matrix which consists of co-factors of each element. In this case, we find the co-factors of each element and enter these values in their corresponding places.

## Adjoint of a Matrix

The adjoint of a square matrix $\mathrm{A}=[$ aij $] \mathrm{nx} \mathrm{n}$ is defined as the transpose of the matrix $[\mathrm{Aij}] \mathrm{n} \times \mathrm{n}$, where Aij are the co-factor of each element aij. It is denoted by Adj A.
In general, adjoint of A is thetranspose of its co-factor matrix.

$$
\text { If } A=\left(\begin{array}{lll}
a_{11} & a_{12} & a_{13} \\
a_{21} & a_{22} & a_{23} \\
a_{31} & a_{32} & a_{33}
\end{array}\right) \text { then Adj } A=\text { Transpose of }\left(\begin{array}{lll}
a_{11} & a_{12} & a_{13} \\
a_{21} & a_{22} & a_{23} \\
a_{31} & a_{32} & a_{33}
\end{array}\right)
$$

$\operatorname{Adj} A=\left(\begin{array}{lll}a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33}\end{array}\right)$

## Important Results

1. If $A$ be any given square matrix of order ' $n$ ' then
$\mathrm{A}(\operatorname{Adj} \mathrm{A})=(\operatorname{Adj} \mathrm{A}) \mathrm{A}=1 \mathrm{AlI}$, where I is the identity matrix of order n
i) A square matrix A is said to be singular if $\mathrm{Al}=0$
ii) A square matrix A is said to be non-singular if 1 Al ? 0
iii) If A is a non-singular matrix of order n the $\operatorname{ladj} \mathrm{Al}=1 \mathrm{Aln}-1$
2. If $A$ and $B$ are nonsingular matrices of the same order, then $A B$ and $B A$ are also non singular matrices of the same order.
3. The determinant of the product of matrices is equal to product of their respective determinants, that is $1 \mathrm{ABl}=1 \mathrm{Al} 1 \mathrm{Bl}$, where A and $B$ are square matrices of same order.
4. A square matrix A is invertible if and only if A is non-singular matrix.

Example: Find the adjoint of $A=\left[\begin{array}{ll}1 & 2 \\ 3 & 4\end{array}\right]$
Solution: $\quad$ Adj $A=\left[\begin{array}{cc}4 & -2 \\ -3 & 1\end{array}\right]$
Adjoint of a $2 \times 2$ matrix is obtained by interchanging the elements of principal diagonal and changing the sign of remaining elements.

## Inverse of a Matrix

If A is a square matrix then its inverse is given by:
$\mathrm{A}^{-1}=\underline{\operatorname{Adj} A}$,
$\mid \mathrm{Al}$ provided A is a non-singular matrix

## Important Result

If $\mathrm{A}-1$ is the inverse of A , then
i) $\mathrm{AA}-1=\mathrm{A}-1 \mathrm{~A}=\mathrm{I}$
ii) $(\mathrm{AB})-1=\mathrm{B}-1 \mathrm{~A}-1$

Example: Find the inverse of $A=\left(\begin{array}{rr}-1 & 2 \\ 0 & 6\end{array}\right)$
$1 \mathrm{Al}=-6-0=-6 ? 0$. So, inverse exists
$\operatorname{Adj} A=\left(\begin{array}{ll}6 & -2 \\ 0 & -1\end{array}\right)$
Hence $A^{-1}=-1 / 6\left(\begin{array}{ll}6 & -2 \\ 0 & -1\end{array}\right)$

Now try it yourself! Should you still need any help, click here to schedule live online session with e Tutor!

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## Reference Links:

- http://en.wikipedia.org/wiki/Cofactor_(linear_algebra)\#Matrix_of_cofactors
- http://www.youtube.com/watch?v=ZMc2WJ1oi-8
- http://en.wikipedia.org/wiki/Transpose
- http://www.britannica.com/EBchecked/topic/561660/square-matrix
- http://en.wikipedia.org/wiki/Identity matrix
- http://en.wikipedia.org/wiki/Determinant
- http://en.wikipedia.org/wiki/Invertible_matrix
- http://www.wikihow.com/Inverse-a-3X3-Matrix

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