ADJOINT AND INVERSE OF A MATRIX

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Co-factors

$A^{-1} = \frac{\text{adj}(A)}{|A|}$ It is a square matrix which consists of co-factors of each element. In this case, we find the co-factors of each element and enter these values in their corresponding places.

Adjoint of a Matrix

The adjoint of a square matrix A = [aij] n x n is defined as the transpose of the matrix [Aij] n x n, where Aij are the co-factor of each element aij. It is denoted by Adj A.
In general, adjoint of A is thetranspose of its co-factor matrix.

$$\text{If } A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \text{ then Adj } A = \text{Transpose of } \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

$$\text{Adj } A = \begin{pmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{pmatrix}$$

Important Results

1. If A be any given square matrix of order ‘n’ then

A (Adj A) = (Adj A) A = lAl I, where I is the identity matrix of order n

i) A square matrix A is said to be singular if lAl=0

ii) A square matrix A is said to be non-singular if lAl?0

iii) If A is a non-singular matrix of order n the ladjAl=lAln-1

2. If A and B are nonsingular matrices of the same order, then AB and BA are also non singular matrices of the same order.

3. The determinant of the product of matrices is equal to product of their respective determinants, that is lABl = lAl lBl, where A and B are square matrices of same order.

4. A square matrix A is invertible if and only if A is non-singular matrix.

Example: Find the adjoint of $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$

Solution: $\text{Adj } A = \begin{pmatrix} 4 & -2 \\ -3 & 1 \end{pmatrix}$

Adjoint of a 2 x 2 matrix is obtained by interchanging the elements of principal diagonal and changing the sign of remaining elements.

Inverse of a Matrix

If A is a square matrix then its inverse is given by:

$$A^{-1} = \frac{\text{Adj } A}{|A|},$$

provided A is a non-singular matrix

Important Result

If A-1 is the inverse of A, then

i) AA-1=A-1A=I

ii) (AB)-1= B-1 A-1

Example: Find the inverse of $A = \begin{pmatrix} -1 & 2 \\ 0 & 6 \end{pmatrix}$

lAl = -6 – 0 = -6 ? 0. So, inverse exists

$$\text{Adj } A = \begin{pmatrix} 6 & -2 \\ 0 & -1 \end{pmatrix}$$

$$\text{Hence } A^{-1} = -1/6 \begin{pmatrix} 6 & -2 \\ 0 & -1 \end{pmatrix}$$

Now try it yourself! Should you still need any help, click here to schedule live online session with e Tutor!

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Reference Links:

- http://en.wikipedia.org/wiki/Cofactor_(linear_algebra)#Matrix_of_cofactors
- http://www.youtube.com/watch?v=ZMc2WJ1oi-8
- http://en.wikipedia.org/wiki/Transpose
- http://www.britannica.com/EBchecked/topic/561660/square-matrix
- http://en.wikipedia.org/wiki/Identity_matrix
- http://en.wikipedia.org/wiki/Determinant
- http://en.wikipedia.org/wiki/Invertible_matrix
- http://www.wikihow.com/Inverse-a-3X3-Matrix

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