## MORE ABOUT MATRICES

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Under this section, we will be learning about important terms which are frequently used in matrices.
We will discuss the following:

- Transpose of a Matrix
- Minors
- Co-factors

Let's study each one in detail.

## Transpose of a Matrix

Let $A$ be a $m x n$ matrix, then its transpose is obtained by interchanging rows into columns. It is denoted by $A^{T}$ or $A^{\prime}$ If $A$ is of order $m \times n$, then the order of $A^{\prime}$ is $n \times m$
For example: $\quad A=\left(\begin{array}{ccc}1 & 4 & 5 \\ 0 & -6 & 9\end{array}\right) \quad$ Order of $A=2 \times 3$

$$
A^{\prime}=\left(\begin{array}{cc}
1 & 0 \\
4 & -6 \\
5 & 9
\end{array}\right) \quad \text { Order of } A^{\prime}=3 \times 2
$$

## Properties of transpose of the Matrix

1) $\left(A^{\prime}\right)^{\prime}=A$
2) $(k A)^{\prime}=k A^{\prime}$
3) $(A+B)^{\prime}=A^{\prime}+B^{\prime}$
4) $(A B)^{\prime}=B^{\prime} A^{\prime}$

Let's try the following examples:

1) If $A=\left[\begin{array}{lll}1 & 4 & 5\end{array}\right]$ then show that $\left(A^{\prime}\right)^{\prime}=A$
Solution: $A=\left[\begin{array}{lll}1 & 4 & 5\end{array}\right]$

$$
A^{\prime}=\left(\begin{array}{l}
1 \\
4 \\
5
\end{array}\right)
$$

$$
\left(A^{\prime}\right)^{\prime}=\left[\begin{array}{lll}
1 & 4 & 5
\end{array}\right]=A
$$

2) If $A=\left(\begin{array}{ccc}-1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1\end{array}\right)$ and $B=\left(\begin{array}{ccc}4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1\end{array}\right)$ then verify that $(A+B)^{\prime}=A^{\prime}+B^{\prime}$
Solution: $A+B=\left(\begin{array}{rrr}-5 & 3 & -2 \\ 6 & 9 & 9 \\ -1 & 4 & 2\end{array}\right)$

$$
(A+B)^{\prime}=\left(\begin{array}{ccc}
-5 & 6 & -1 \\
3 & 9 & 4 \\
-2 & 9 & 2
\end{array}\right) \ldots \text { (i) }
$$

$$
A^{\prime}=\left(\begin{array}{ccc}
-1 & 5 & -2 \\
2 & 7 & 1 \\
3 & 9 & 1
\end{array}\right) \quad B^{\prime}=\left(\begin{array}{ccc}
-4 & 1 & 1 \\
1 & 2 & 3 \\
-5 & 0 & 1
\end{array}\right)
$$

$$
A^{\prime}+B^{\prime}=\left(\begin{array}{ccc}
-5 & 6 & -1 \\
3 & 9 & 4 \\
-2 & 9 & 2
\end{array}\right) \ldots \text { (ii) }
$$

From (i) and (ii), we get, $(\mathrm{A}+\mathrm{B})^{\prime}=\mathrm{A}^{\prime}+\mathrm{B}^{\prime}$

## Minor of an element

Minor of an element $\mathrm{a}_{\mathrm{ij}}$ of a determinant is the determinant obtained by deleting its $\mathrm{i}^{\text {th }}$ row and $\mathrm{j}^{\text {th }}$ column in which $\mathrm{a}_{\mathrm{ij}}$ lies. It is denoted by $\mathrm{M}_{\mathrm{ij}}$.
Minor of an element of a determinant of order $n(n ? 2)$ is a determinant of order $n-1$
Example: Find the minor of the element 3 in the determinant
$\Delta=\left|\begin{array}{lll}1 & 2 & 3 \\ 0 & 5 & 6 \\ -3 & 4 & 7\end{array}\right|$

Solution: The element 3 lies in first row and third column, so it is denoted by $M^{13}$ and is given by $\left.\quad$| $M_{13}=$ | 0 |
| :---: | :---: |
|  | 5 |
| -3 | 4 | \right\rvert\,

[Deleting 1st row and 3rd column]

$$
\begin{aligned}
& =0-(-15) \\
& =15
\end{aligned}
$$

## Co-factor of an element

Co-factor of an element aij denoted by $\mathrm{A}_{\mathrm{ij}}$ is defined by $\mathrm{A}_{\mathrm{ij}}=(-1) \mathrm{i}+\mathrm{j} \mathrm{M}_{\mathrm{ij}}$, where $\mathrm{M}_{\mathrm{ij}}$ is the minor of $\mathrm{a}_{\mathrm{ij}}$.
Example: Find the co-factor of element -5 in the determinant $\Delta=\left|\begin{array}{ccc}2 & -1 & 3 \\ -1 & 5 & 0 \\ -5 & 3 & 6\end{array}\right|$

Solution: -5 belongs to 3 rd row and 1 st column, so it is denoted by
$A_{31}=(-1)^{3+1}\left|\begin{array}{rr}-1 & 3 \\ 5 & 0\end{array}\right|$

$$
\begin{aligned}
& =+(0-15) \\
& =-15
\end{aligned}
$$

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## Reference Links:

- http://en.wikipedia.org/wiki/Transpose
- http://www.mathreference.com/la-mpoly,order.html
- http://en.wikipedia.org/wiki/Minor_(linear_algebra)
- http://en.wikipedia.org/wiki/Determinant
- http://en.wikipedia.org/wiki/Cofactor_(linear_algebra)


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