MULTIPLICATION OF MATRICES

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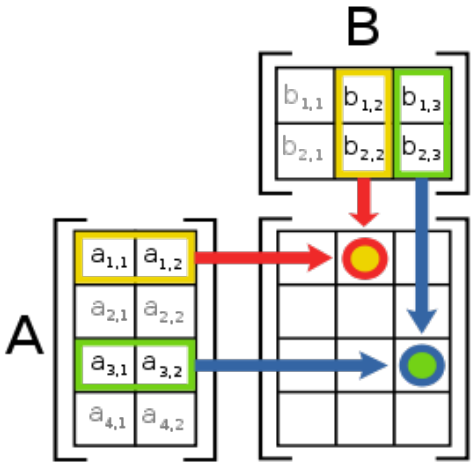
Multiplication of a Matrix by a Scalar



When a matrix is multiplied by a scalar then each element of that matrix is multiplied by the scalar. In general, we can say, if A = [aij]m x n is a matrix and 'k' is a scalar, then kA is another matrix which is obtained by multiplying each element of A by 'k'.

For example: If $A = \begin{bmatrix} 2 & 3 & 4 \\ 0 & -7 & 1 \end{bmatrix}$

then, 5A is obtained by multiplying each element by 5

$5A = \begin{bmatrix} 10 & 15 & 20 \\ 0 & -35 & 5 \end{bmatrix}$

Properties of Scalar Multiplication

i) If A and B are matrices and k is a scalar then k(A + B) = kA + kB
ii) If A is a matrix and k and l are scalars then (k + l) A = kA + lA

Multiplication of Matrices

The product of two matrices A and B is defined if the number of columns of A is equal to the number of rows of B. Let A = [aij] be m x n matrix and B = [bjk] be an n x p matrix. Then, the product of A and B is a matrix C of order m x p.

For example: Find AB, if $A = \begin{bmatrix} 3 & -2 \\ 4 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -3 & 4 \\ 0 & 2 & 5 \end{bmatrix}$

Solution: Matrix A has 2 columns and B has 2 rows, so number of columns of 1st matrix is same as number of rows of 2nd matrix, hence it is conformable for multiplication.

$$AB = \begin{bmatrix} 3 \times 1 + (-2) \times 0 & 3 \times (-3) + (-2) \times 2 & 3 \times 4 + (-2) \times 5 \\ 4 \times 1 + 7 \times 0 & 4 \times (-3) + 7 \times 2 & 4 \times 4 + 7 \times 5 \end{bmatrix}$$

$$= \begin{bmatrix} 3 - 0 & -9 - 4 & 12 - 10 \\ 4 + 0 & -12 + 14 & 16 + 35 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -13 & 2 \\ 4 & 2 & 51 \end{bmatrix}$$

Order of A = 2 x 2 and order of B = 2 x 3, so order of AB = 2 x 3

Properties of Multiplication of Matrices

i) Associative Law: For any three matrices A, B and C, we have
(AB)C = A(BC), whenever both sides of equality are defined.
ii) Distributive law: For three matrices A, B and C
a) A (B + C) = AB + AC
b) (A + B) C = AC + BC

iii) Existence of multiplicative identity: For every square matrix A, there exists an identity matrix of same order such that IA = AI = A

For example: Find $A^2 - 5A + 6I$, if $A = \begin{pmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{pmatrix}$

$$A^2 = \begin{pmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{pmatrix} \times \begin{pmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 4+0+1 & 0+0-1 & 2+0+0 \\ 4+2+3 & 0+1-3 & 2+3+0 \\ 2-2+0 & 0-1-0 & 1-3+0 \end{pmatrix}$$

$$= \begin{pmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{pmatrix}$$

$$5A = \begin{pmatrix} 10 & 0 & 5 \\ 10 & 5 & 15 \\ 5 & -5 & 0 \end{pmatrix}$$

$$6I = \begin{pmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{pmatrix}$$

$$A^2 - 5A + 6I = \begin{pmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{pmatrix} - \begin{pmatrix} 10 & 0 & 5 \\ 10 & 5 & 15 \\ 5 & -5 & 0 \end{pmatrix} + \begin{pmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & -1 & -3 \\ -1 & -1 & -10 \\ -5 & 4 & 4 \end{pmatrix}$$

iv) Matrix multiplication is not commutative. If A and B are any two matrices then AB?BA

Now try it yourself! Should you still need any help, click here to schedule live online session with e Tutor!

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Reference Links:

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- http://dictionary.reference.com/browse/multiplicative+identity

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